MATHEMATICS · GRADES 10-12

Angles between vectors

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how engineers and game designers work out the angle between two forces pushing on something, or between the direction a character is facing and the direction it needs to move. Two pushes at an angle to each other do not add up the way two pushes in the same direction do. The angle between them tells you how much they help each other and how much they fight each other.



Rule

Angle between vectors: cos of the angle = $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| \times |\mathbf{b}|}$, where a · b is the dot product of the two vectors and |a| and |b| are their lengths.

The dot product is the sum of the products of matching components, and you find the angle by taking the inverse cosine of the result.

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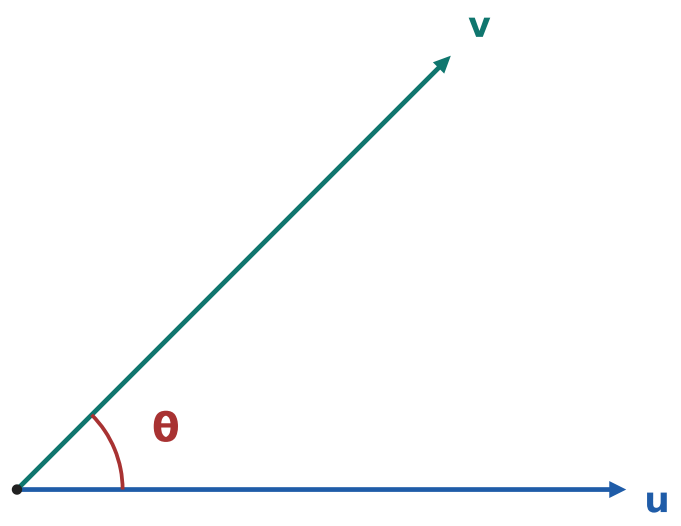
Worked examples

EXAMPLE 1

Q Find the angle between (1, 0) and (0, 1).



SOLUTION

Dot product: $1 \times 0 + 0 \times 1 = 0$

Both magnitudes are 1, so cos of the angle = $\frac{0}{1} = 0$

Angle = 90 degrees.

EXAMPLE 2

Q Find the angle between (1, 0) and (1, 1).

SOLUTION

Dot product: $1 \times 1 + 0 \times 1 = 1$

Magnitudes: 1 and root 2, so cos = $\frac{1}{\text{root } 2}$, about 0.707

Angle = 45 degrees.

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Practice questions

1. A drone flies east in the direction (1, 0), then keeps flying east in the direction (1, 0). What is the angle between its two flight directions?

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2. A boat sails east in the direction (1, 0), then turns and sails west in the direction (-1, 0). What is the angle between the two directions?

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3. One force pushes east in the direction (2, 0) and another pushes north in the direction (0, 3). What is the angle between the forces?

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4 One force pushes in the direction (1, 0) and another pushes in the direction (1, root 3). What is the angle between the forces?

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5 Which formula gives cos of the angle between two vectors u and v?

A cos of the angle = $\frac{(u \text{ dot } v)}{(|u|\ |v|)}$

B cos of the angle = $\frac{(u \text{ dot } v)}{(|u| + |v|)}$

C sin of the angle = $\frac{(u \text{ dot } v)}{(|u|\ |v|)}$

D cos of the angle = $\frac{(u \text{ cross } v)}{(|u|\ |v|)}$

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6 A ramp runs in the direction (3, 0) and a rail runs in the direction (3, 3). What is the angle between them, to the nearest degree?

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7 Two vectors have a dot product of 0. What is the angle between them?

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8 A hiker walks east in the direction (1, 0), then turns and walks south in the direction (0, -1). What is the angle between the two legs of the hike?

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9 A ski slope drops in the direction (2, 2) while a lift cable runs in the direction (2, 0). What is the angle between them?

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10 The dot product of two vectors is positive. Is the angle between them acute or obtuse?

A Acute

B Obtuse

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ANSWER KEY

1. 0 degrees

2. 180 degrees

3. 90 degrees

4. 60 degrees

5. cos of the angle = $\frac{(u \text{ dot } v)}{(|u|\ |v|)}$

6. 45 degrees

7. 90 degrees

8. 90 degrees

9. 45 degrees

10. Acute

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