MATHEMATICS · GRADES 10-12

Basic substitution integration

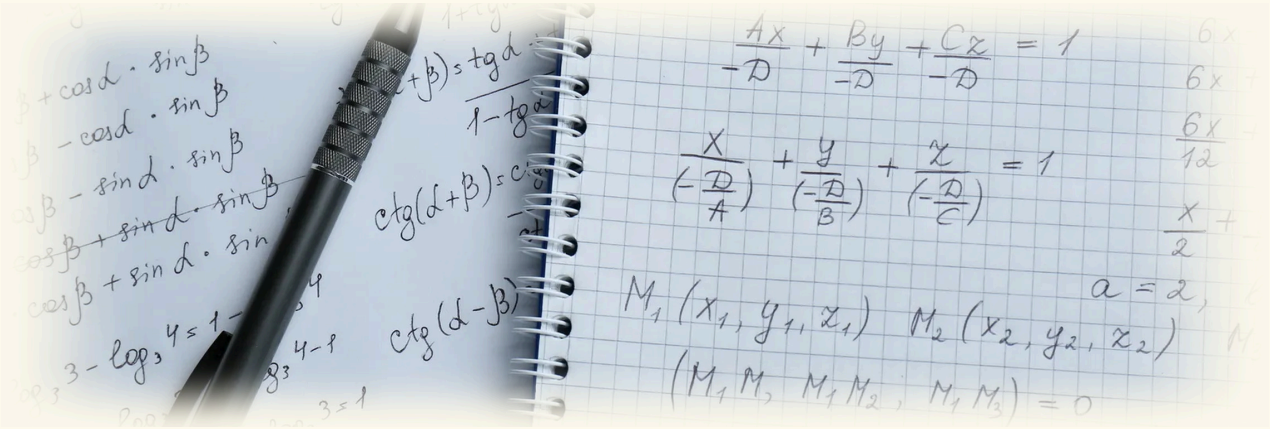
Difficulty level: Advanced

Mathematics: Grades 10-12

This document was made by AI and there may be mistakes. If a question or answer looks wrong, use the Flag a mistake button to let Brett know so he can fix it.

Overview of the concept

This is how mathematicians simplify a hard total by swapping in a simpler stand-in. It is the same trick engineers use when a messy formula hides a simple pattern inside it. Spotting the pattern and substituting for it turns a problem that looks impossible into one you already know how to solve.



Rule

Substitution: let u be the inside function.

If the derivative of u also appears in the integral, replace everything with u, integrate, then put the original expression back in place of u and add C.

© 2026 Brett Robbins. All Rights Reserved.

Worked examples

EXAMPLE 1

Q Integrate $2x \times (x^2 + 1)$.

SOLUTION

Let $u = x^2 + 1$. Then $du/dx = 2x$, which means $2x\,dx$ is the same as du. So the $2x$ in front, together with the dx, becomes du.

The integral becomes the integral of u, which is $\frac{u^2}{2}$.

Substitute back: $\frac{(x^2 + 1)^2}{2} + C$

EXAMPLE 2

Q Integrate $3x^2 \times (x^3 + 2)$.

SOLUTION

Let $u = x^3 + 2$. Then $du/dx = 3x^2$, which means $3x^2\,dx$ is the same as du. So the $3x^2$ in front, together with the dx, becomes du.

The integral becomes the integral of u, which is $\frac{u^2}{2}$.

Substitute back: $\frac{(x^3 + 2)^2}{2} + C$

© 2026 Brett Robbins. All Rights Reserved.

Practice questions

1 Integrate $2x \times (x^2 + 3)$, in terms of u first.

- **A** $(x^2 + 3)^2 + C$
- **B** $\frac{(x^2 + 3)^2}{2} + C$
- **C** $\frac{(x^2 + 3)^3}{3} + C$
- **D** $2(x^2 + 3)^2 + C$

Understand It with Pulse | Flag a mistake

2 What do you choose for u?

- **A** $u = 2x$
- **B** $u = x^2$
- **C** $u = x^2 + 3$
- **D** $u = 3x$

Understand It with Pulse | Flag a mistake

© 2026 Brett Robbins. All Rights Reserved.

3 Integrate $(x + 1)^2 \times 1$, by letting $u = x + 1$.

A $\frac{(x + 1)^3}{3} + C$

B $(x + 1)^3 + C$

C $\frac{(x + 1)^2}{2} + C$

D $\frac{(x + 1)^4}{4} + C$

Understand It with Pulse | Flag a mistake

4 Integrate $4x \times (x^2 + 1)$.

A $2(x^2 + 1)^2 + C$

B $\frac{(x^2 + 1)^3}{3} + C$

C $\frac{(x^2 + 1)^2}{2} + C$

D $(x^2 + 1)^2 + C$

Understand It with Pulse | Flag a mistake

© 2026 Brett Robbins. All Rights Reserved.

5 Why must the derivative of u appear in the integral?

- **A** So that u stays in the final answer
- **B** So that the derivative can be integrated separately
- **C** So that du can replace the rest of the integral
- **D** So that x can be replaced by a constant

Understand It with Pulse | Flag a mistake

6 Integrate $(2x + 1) \times (x^2 + x)$.

- **A** $\frac{(x^2 + x)^2}{2} + C$
- **B** $(x^2 + x)^2 + C$
- **C** $\frac{(x^2 + x)^3}{3} + C$
- **D** $2(x^2 + x)^2 + C$

Understand It with Pulse | Flag a mistake

© 2026 Brett Robbins. All Rights Reserved.

7 What is the last step of substitution?

A Differentiate u with respect to x

B Substitute back from u to x

C Add the constant C first

D Replace x with u again

Understand It with Pulse | Flag a mistake

8 Integrate $6x^2 \times (x^3 + 1)$.

A $\frac{(x^3 + 1)^2}{2} + C$

B $(x^3 + 1)^2 + C$

C $2(x^3 + 1)^2 + C$

D $\frac{(x^3 + 1)^3}{3} + C$

Understand It with Pulse | Flag a mistake

© 2026 Brett Robbins. All Rights Reserved.

9 Integrate $3 \times (3x + 1)^2$, by letting $u = 3x + 1$.

A $(3x + 1)^3 + C$

B $\frac{(3x + 1)^2}{2} + C$

C $3(3x + 1)^3 + C$

D $\frac{(3x + 1)^3}{3} + C$

Understand It with Pulse | Flag a mistake

10 Integrate $x \times (x^2 + 4)$.

A $\frac{(x^2 + 4)^2}{4} + C$

B $\frac{(x^2 + 4)^2}{2} + C$

C $(x^2 + 4)^2 + C$

D $\frac{(x^2 + 4)^3}{3} + C$

Understand It with Pulse | Flag a mistake

© 2026 Brett Robbins. All Rights Reserved.

ANSWER KEY

1. $\frac{(x^2 + 3)^2}{2} + C$

2. $u = x^2 + 3$ (the inside function, from the previous question)

3. $\frac{(x + 1)^3}{3} + C$

4. $(x^2 + 1)^2 + C$

5. So that du can replace the rest of the integral

6. $\frac{(x^2 + x)^2}{2} + C$

7. Substitute back from u to x

8. $(x^3 + 1)^2 + C$

9. $\frac{(3x + 1)^3}{3} + C$

10. $\frac{(x^2 + 4)^2}{4} + C$

© 2026 Brett Robbins. All Rights Reserved.