MATHEMATICS · GRADES 10-12

Chain rule

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you find the rate of change of something that changes in stages. A balloon grows as you blow air in, and how fast it grows depends on how fast the air goes in, which is one change feeding into the next. The chain rule links those stages together so you can work out the overall rate in one go.



Rule

Chain rule: the derivative of f(g(x)) is f'(g(x)) × g'(x). Differentiate the outside, then multiply by the derivative of the inside.

When a function is built in stages, you work from the outside in.

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Worked examples

EXAMPLE 1

Q Differentiate $(x + 1)^2$.

SOLUTION

Outside: square, so its derivative is 2 times the inside: $2(x + 1)$.

Inside: $x + 1$, whose derivative is 1.

Multiply: $2(x + 1) \times 1 = 2x + 2$

EXAMPLE 2

Q Differentiate $(3x + 2)^2$.

SOLUTION

Outside derivative: $2(3x + 2)$

Inside derivative: 3

Multiply: $2(3x + 2) \times 3 = 18x + 12$

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Practice questions

1 State the chain rule.

- **A** $d/dx\ f(g(x)) = f'(g(x)) \times g'(x)$
- **B** $d/dx\ f(g(x)) = f'(g(x))$
- **C** $d/dx\ f(g(x)) = f'(x) \times g'(x)$
- **D** $d/dx\ f(g(x)) = g'(f(x)) \times f'(x)$

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2 Differentiate $(x + 5)^2$

- **A** $2x + 5$
- **B** $x + 5$
- **C** $2(x + 5)$
- **D** $2(x + 5)^2$

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3 Differentiate $(2x + 1)^2$

- **A** $2(2x + 1)$
- **B** $4(2x + 1)$
- **C** $4x + 2$
- **D** $2x + 1$

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4 Differentiate $(x^2 + 1)^3$. Give the outer step only.

- **A** $3(x^2 + 1)^2$
- **B** $6x(x^2 + 1)^2$
- **C** $3(x^2 + 1)$
- **D** $2x(x^2 + 1)^2$

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5 Differentiate $(4x)^2$

A $8x$

B $16x$

C $16x^2$

D $32x$

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6 Differentiate $(x + 1)^3$

A $(x + 1)^2$

B $3(x + 1)^2$

C $3(x + 1)^3$

D $3x + 3$

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7 What is the inside function of $(3x + 1)^2$?

A $3x$

B $x + 1$

C $3x + 1$

D $(3x + 1)^2$

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8 Differentiate $(5x + 2)^2$

A $2(5x + 2)$

B $10x + 2$

C $5(5x + 2)$

D $10(5x + 2)$

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9 Differentiate $(x - 3)^2$

- **A** 2(x - 3)
- **B** 2(x + 3)
- **C** x - 3
- **D** 2x - 3

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10 Why multiply by the inside derivative?

- **A** Because the outside is also changing with x
- **B** Because the inside is also changing with x
- **C** Because the derivative of a constant is always zero
- **D** Because you always multiply by 2

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ANSWER KEY

1. $d/dx\ f(g(x)) = f'(g(x)) \times g'(x)$

2. $2(x + 5)$

3. $4(2x + 1)$

4. $3(x^2 + 1)^2$

5. $32x$

6. $3(x + 1)^2$

7. $3x + 1$

8. $10(5x + 2)$

9. $2(x - 3)$

10. Because the inside is also changing with x

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