MATHEMATICS · GRADES 10-12

Change-of-base formula

Difficulty level: Advanced

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Overview of the concept

This is how you work out a logarithm your calculator does not have a button for. Most calculators only have base ten and natural log buttons, so any other base has to be rewritten with one of those. The change-of-base formula is that rewrite, and it is what lets you solve log problems with any base on an ordinary calculator.



Rule

Change of base: log base b of x = $\dfrac{\log x}{\log b}$.

You can rewrite a logarithm in any base you like, using common logs or natural logs on top and bottom. The new base must be the same on the top and the bottom.

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Worked examples

EXAMPLE 1

Q Evaluate $\log_2(8)$ using change of base

SOLUTION

Write it with base ten logs: $\log_2(8) = \frac{\log(8)}{\log(2)}$.

$\log 8 \div \log 2 = 0.903 \div 0.301 = 3$

So $\log_2(8) = 3$, which matches $2^3 = 8$.

EXAMPLE 2

Q Evaluate $\log_5(25)$ using change of base

SOLUTION

Write it with natural logs: $\log_5(25) = \frac{\ln(25)}{\ln(5)}$.

$\ln 25 \div \ln 5 = 3.219 \div 1.609 = 2$

So $\log_5(25) = 2$, which matches $5^2 = 25$.

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Practice questions

1 Evaluate $\log_2(16)$ using change of base.

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2 Evaluate $\log_3(81)$ using change of base.

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3 Evaluate $\log_4(64)$ using change of base.

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4 Evaluate $\log_5(125)$ using change of base.

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5 Evaluate $\log_{10}(1000)$ using change of base.

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6 Evaluate $\log_2(64)$ using change of base.

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7 One of these logarithms equals 3. Which one?

A $\log_3(27)$

B $\log_3(9)$

C $\log_4(8)$

D $\log_5(25)$

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8 Change of base works for any new base. Why?

- **A** A logarithm is the power you raise the base to, so the ratio of two logarithms with the same new base gives that power
- **B** Change of base only works for base 10
- **C** Change of base only works when the new base is e
- **D** Change of base changes the value of the logarithm

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9 Evaluate $\log_2(32)$ using change of base.

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10 Three of these logarithms equal 4 and one does not. Which one is not 4?

A $\log_2(16)$

B $\log_3(81)$

C $\log_4(64)$

D $\log_5(625)$

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ANSWER KEY

1. 4

2. 4

3. 3

4. 3

5. 3

6. 6

7. $\log_3(27)$

8. A logarithm is the power you raise the base to, so the ratio of two logarithms with the same new base gives that power

9. 5

10. $\log_4(64)$

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