MATHEMATICS · GRADES 10-12

Completing the square for circle equations

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you recover the centre and radius of a circle from a messy expanded equation. A mapmaker recovers the centre and reach of a circular zone from its coordinates the same way. Completing the square is the tool that turns a jumble of terms back into a circle you can actually picture and work with.



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Worked examples

EXAMPLE 1

Q **Find the centre and radius of $x^2 + y^2 - 6x = 0$.**

SOLUTION

Look at the x terms: $x^2 - 6x$. Take half of 6, which is 3, and square it: $3^2 = 9$. That 9 completes the square, so $x^2 - 6x = (x - 3)^2 - 9$: the square $(x - 3)^2$ carries a +9 inside it, so the − 9 takes it back out to keep the value the same.

So $(x - 3)^2 - 9 + y^2 = 0$

So $(x - 3)^2 + y^2 = 9$: centre $(3, 0)$, radius 3.

EXAMPLE 2

Q **Find the centre and radius of $x^2 + y^2 + 4y = 0$.**

SOLUTION

Complete the square in y: half of 4 is 2, and $2^2 = 4$, so $y^2 + 4y = (y + 2)^2 - 4$.

So $x^2 + (y + 2)^2 = 4$

Centre $(0, -2)$, radius 2

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Practice questions

1 $x^2 + y^2 - 4x = 0$. What are its centre and radius?

- **A** Centre (2, 0), radius 2
- **B** Centre (-2, 0), radius 2
- **C** Centre (2, 0), radius 4
- **D** Centre (0, 2), radius 2

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2 Which point lies on the circle $x^2 + y^2 - 6x = 0$?

- **A** (1, 0)
- **B** (0, 0)
- **C** (3, 1)
- **D** (0, 3)

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3 $(x - 1)^2 + y^2 = 16$. What are its centre and radius?

- **A** Centre (-1, 0), radius 4
- **B** Centre (1, 0), radius 16
- **C** Centre (1, 0), radius 4
- **D** Centre (0, 1), radius 4

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4 Why do you complete the square on a circle equation?

- **A** To rewrite an expanded circle equation in centre-radius form
- **B** To find the area of a circle
- **C** To make the radius bigger
- **D** To remove the centre from the equation

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5 $x^2 + y^2 - 4y = 12$. What is the centre of the circle?

- **A** (2, 0)
- **B** (0, -2)
- **C** (0, 2)
- **D** (0, 4)

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6 $x^2 + y^2 - 2x - 2y = 0$. What are its centre and radius?

- **A** Centre (1, 1), radius 2
- **B** Centre (1, 1), radius $\sqrt{2}$
- **C** Centre (-1, -1), radius $\sqrt{2}$
- **D** Centre (1, 1), radius 1

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7 $x^2 + y^2 + 8x = 0$. What is its radius?

- **A** 8
- **B** 16
- **C** 4
- **D** 2

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8 Complete the square to rewrite $x^2 + 8x$. What do you get?

- **A** $(x + 4)^2 - 16$
- **B** $(x + 4)^2 + 16$
- **C** $(x - 4)^2 - 16$
- **D** $(x + 8)^2 - 64$

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9 Write $x^2 + y^2 - 6x + 5 = 0$ in centre-radius form.

- **A** $(x + 3)^2 + y^2 = 4$
- **B** $(x - 3)^2 + y^2 = 9$
- **C** $(x - 3)^2 + y^2 = 14$
- **D** $(x - 3)^2 + y^2 = 4$

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10 $x^2 + y^2 + 2x + 2y + 1 = 0$. What are its centre and radius?

- **A** Centre (1, 1), radius 1
- **B** Centre (-1, -1), radius 2
- **C** Centre (1, -1), radius 1
- **D** Centre (-1, -1), radius 1

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ANSWER KEY

1. Centre (2, 0), radius 2

2. (0, 0)

3. Centre (1, 0), radius 4

4. To rewrite an expanded circle equation in centre-radius form

5. (0, 2)

6. Centre (1, 1), radius $\sqrt{2}$

7. 4

8. $(x + 4)^2 - 16$

9. $(x - 3)^2 + y^2 = 4$

10. Centre (-1, -1), radius 1

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