MATHEMATICS · GRADES 10-12

Completing the square

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you solve a quadratic that will not factor neatly, like finding when a thrown ball hits the ground. You rewrite it as a perfect square you can undo with a square root, which gives you the answer even when factoring fails. It is the method behind the quadratic formula, and it works on every quadratic, not just the easy ones.



Rule

Completing the square: x^2 plus bx equals $(x \text{ plus } \frac{b}{2})^2$ minus $(\frac{b}{2})^2$.

Adding $(\frac{b}{2})^2$ to x^2 plus bx makes a perfect square. That lets you rewrite a quadratic as a square you can undo with a square root.

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Worked examples

EXAMPLE 1

Q Solve $x^2 + 6x + 5 = 0$ by completing the square

SOLUTION

Move the constant: $x^2 + 6x = -5$

$(6 \div 2)^2 = 3^2 = 9$

$x^2 + 6x + 9 = -5 + 9 = 4$

Write as a square and solve: $(x + 3)^2 = 4$, so $x + 3 = 2$ or $x + 3 = -2$, giving $x = -1$ or $x = -5$

EXAMPLE 2

Q Solve $x^2 - 8x + 7 = 0$ by completing the square

SOLUTION

Move the constant: $x^2 - 8x = -7$

$(-8 \div 2)^2 = (-4)^2 = 16$

$x^2 - 8x + 16 = -7 + 16 = 9$

Write as a square and solve: $(x - 4)^2 = 9$, so $x - 4 = 3$ or $x - 4 = -3$, giving $x = 7$ or $x = 1$

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Practice questions

1 Solve $x^2 + 4x + 3 = 0$ by completing the square.

- **A** x = 1 or x = 3
- **B** x = -1 or x = 3
- **C** x = -1 or x = -3
- **D** x = 1 or x = -3

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2 Solve $x^2 - 6x + 5 = 0$ by completing the square.

- **A** x = -1 or x = 5
- **B** x = 1 or x = 5
- **C** x = -1 or x = -5
- **D** x = 1 or x = -5

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3 What number must be added to $x^2 + 10x$ to make a perfect square?

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4 Solve $x^2 - 2x - 8 = 0$ by completing the square.

A. $x = 4$ or $x = -2$

B. $x = -4$ or $x = 2$

C. $x = 4$ or $x = 2$

D. $x = -4$ or $x = -2$

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5 What is the minimum value of $x^2 + 8x + 12$?

- **A** -4
- **B** 4
- **C** -2
- **D** 16

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6 Solve $x^2 + 6x - 16 = 0$ by completing the square.

- **A** x = -2 or x = 8
- **B** x = 2 or x = -8
- **C** x = 2 or x = 8
- **D** x = -2 or x = -8

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7 A quadratic has solutions $x = 1$ and $x = -7$. Which of these is it?

- **A** $x^2 - 6x - 7 = 0$
- **B** $x^2 + 6x + 7 = 0$
- **C** $x^2 + 6x - 7 = 0$
- **D** $x^2 - 6x + 7 = 0$

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8 Nia solves $x^2 + 8x + 12 = 0$ by completing the square. She writes $(x + 4)^2 = 4$, then $x + 4 = 2$, so $x = -2$. What did she miss?

- **A** She added 9 to only one side
- **B** She forgot the negative square root
- **C** She should have divided by 2 first
- **D** She took the square root of 4 wrongly

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9 Write $x^2 + 12x + 32$ as a completed square.

A $(x - 6)^2 - 4$

B $(x + 6)^2 + 4$

C $(x + 6)^2 - 4$

D $(x + 6)^2 - 36$

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10 Three of these equations have $x = -1$ as a solution and one does not. Which one does not:

A $x^2 + 4x + 3 = 0$

B $x^2 + 6x + 5 = 0$

C $x^2 + 8x + 15 = 0$

D $x^2 + 5x + 4 = 0$

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ANSWER KEY

1. $x = -1$ or $x = -3$

2. $x = 1$ or $x = 5$

3. 25

4. $x = 4$ or $x = -2$

5. -4

6. $x = 2$ or $x = -8$

7. $x^2 + 6x - 7 = 0$

8. She forgot the negative square root

9. $(x + 6)^2 - 4$

10. $x^2 + 8x + 15 = 0$

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