MATHEMATICS · GRADES 10-12

Cosine law

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you work out a distance or an angle in a triangle that is not a right triangle. A surveyor uses it to measure across a lake or a river without walking through the water. A builder uses it to find the angle of a roof that is not square. Any time you know two sides and the angle between them, or all three sides, it gives you the missing piece.



Rule

Cosine law: $a^2 = b^2 + c^2 - 2bc \cos A$.

It gives a side of any triangle from the other two sides and the angle between them. When the angle is 90 degrees, cos A is 0 and it becomes the Pythagorean theorem.

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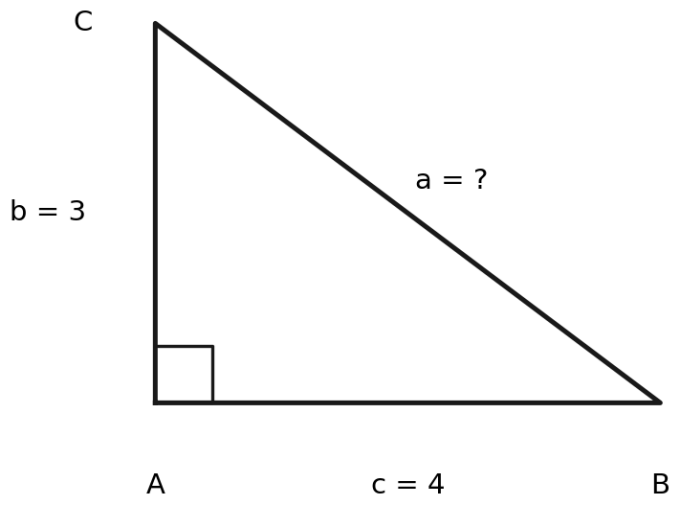
Worked examples

EXAMPLE 1

Q Sides b = 3, c = 4, included angle A = 90 degrees. Find a.



SOLUTION

Cosine law: $a^2 = b^2 + c^2 - 2bc \cos A$

Cos 90 degrees = 0, so $a^2 = 9 + 16 - 0 = 25$

So a = 5, which matches the Pythagorean theorem.

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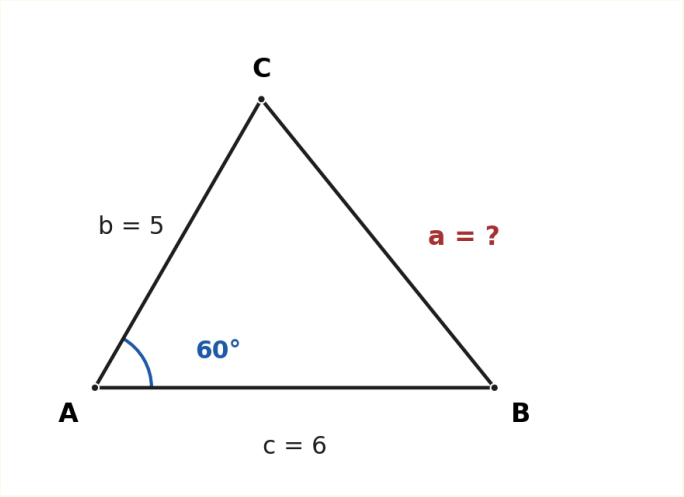
EXAMPLE 2

Q Sides b = 5, c = 6, included angle A = 60 degrees. Find a.



SOLUTION

$a^2 = 25 + 36 - 2 \times 5 \times 6 \times \cos 60$ degrees

Cos 60 degrees = 0.5, so $a^2 = 61 - 30 = 31$

So $a = \sqrt{31}$, about 5.57

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Practice questions

1 A triangle has sides a, b, c and angle A opposite side a. Write the cosine law for side a.

- **A** $a^2 = b^2 + c^2 - 2bc \cos A$
- **B** $a^2 = b^2 + c^2 + 2bc \cos A$
- **C** $a^2 = b^2 + c^2 - 2bc \sin A$
- **D** $a^2 = b^2 - c^2 - 2bc \cos A$

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2 $b = 6$, $c = 8$, and the angle between them $A = 90$ degrees. Find side a.

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3 $b = 3$, $c = 3$, and $A = 60$ degrees. Find side a.

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4 $b = 5$, $c = 12$, and $A = 90$ degrees. Find the third side, a.

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5 In what situation is the cosine law the right choice?

- **A** When you know two sides and the included angle, or all three sides
- **B** When you know two angles and one side
- **C** When you know one side and the angle opposite it
- **D** When you know the area and the perimeter

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6 b = 4, c = 4, and A = 60 degrees. Find side a.

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7 b = 2, c = 2, and A = 90 degrees. Find side a, to two decimal places.

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8 A triangle has a right angle, A = 90 degrees. What does the cosine law simplify to?

A. The Pythagorean theorem

B. The sine law

C. The tangent rule

D. The area formula

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9 b = 7, c = 24, and A = 90 degrees. Find side a.

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10 b = 10, c = 10, and A = 60 degrees. Find side a.

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ANSWER KEY

1. $a^2 = b^2 + c^2 - 2bc \cos A$

2. 10

3. 3

4. 13

5. When you know two sides and the included angle, or all three sides

6. 4

7. 2.83

8. The Pythagorean theorem

9. 25

10. 10

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