MATHEMATICS · GRADES 10-12

Critical points

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you find the highest or lowest point of something that changes. A shop owner uses it to find the price that brings in the most profit, not so high that nobody buys and not so low that the profit vanishes. A runner uses it to find the fastest pace they can hold without burning out. Any time a quantity rises then falls, or falls then rises, the turning point is where the best answer sits.



Rule

Critical points are where the derivative is 0.

At a critical point the graph is flat for an instant, so it is where a maximum or minimum can happen. Find them by differentiating, setting the derivative equal to 0, and solving for x.

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Worked examples

EXAMPLE 1

Q Find the critical points of $f(x) = x^2$.

SOLUTION

Derivative: $2x$

Set it to 0: $2x = 0$, so $x = 0$

Critical point: $x = 0$

EXAMPLE 2

Q Find the critical points of $f(x) = x^3 - 3x$.

SOLUTION

Derivative: $3x^2 - 3$

Set to 0: $3x^2 = 3$, so $x^2 = 1$, so $x = 1$ or $x = -1$

Critical points: $x = 1$ and $x = -1$

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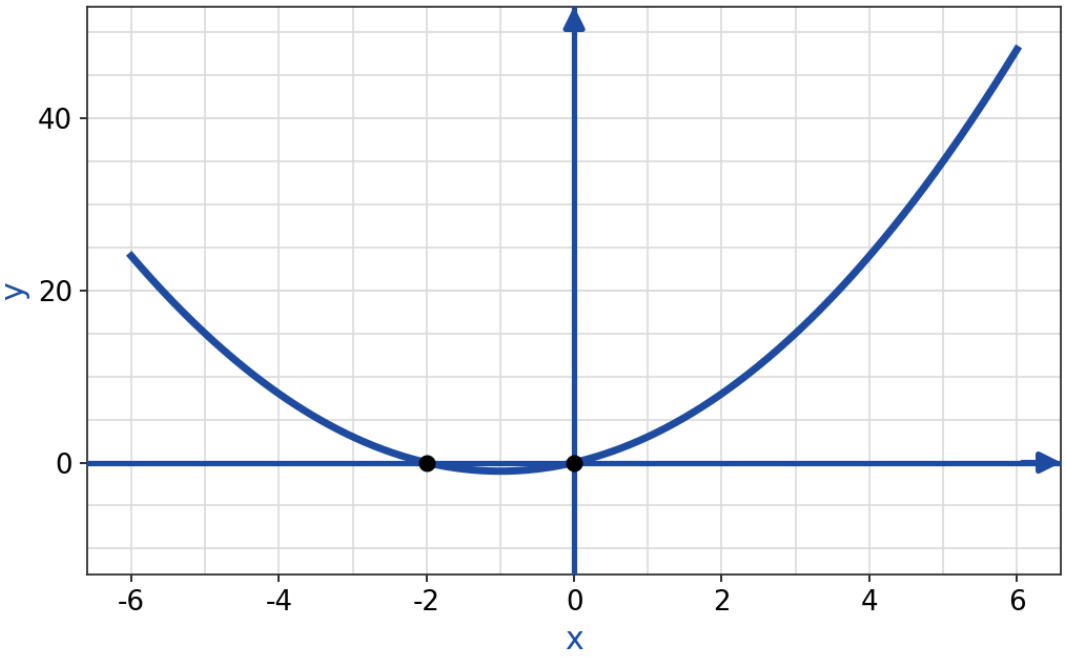
Practice questions

1 Where is the critical point of $f(x) = x^2 + 2x$?



- **A** x = 1
- **B** x = -1
- **C** x = 0
- **D** x = 2

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2 Where are the critical points of $f(x) = x^3 - 3x^2$?

- **A** x = 0 and x = -2
- **B** x = 1 and x = 2
- **C** x = 0 and x = 2
- **D** x = -1 and x = 2

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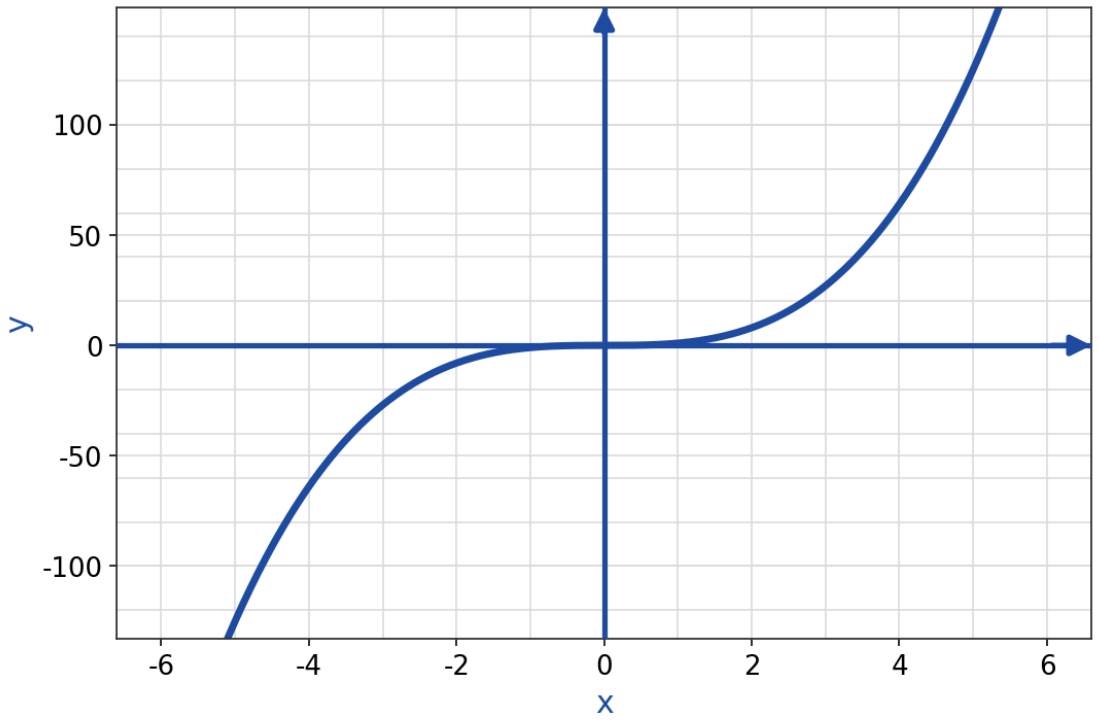
3 The volume of water is $f(x) = x^3$, where x is the time in minutes. At what time is the flow rate zero for an instant?



A x = 1

B x = 0

C x = -1

D x = 3

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4 **Find the stationary points of $f(x) = x^3 - 6x^2 + 9x$.**

- **A** x = 1 and x = -3
- **B** x = -1 and x = 3
- **C** x = 0 and x = 3
- **D** x = 1 and x = 3

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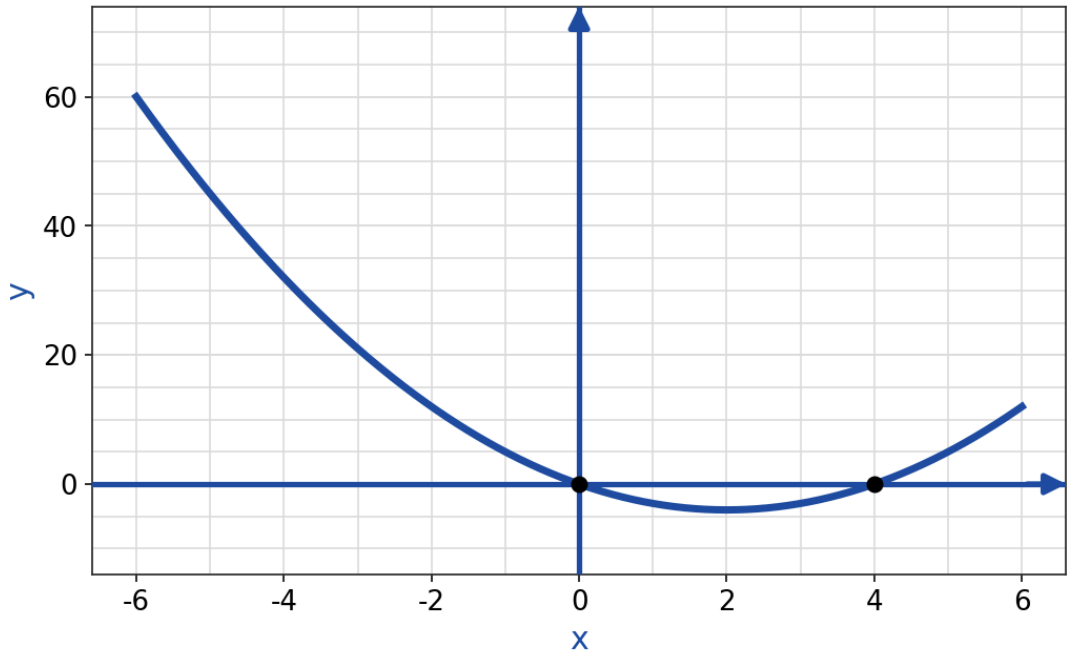
5 For $f(x) = x^2 - 4x$, at what value of x is the graph at its lowest?



- **A** x = -2
- **B** x = 2
- **C** x = 0
- **D** x = 4

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6 **Find the stationary points of $f(x) = x^3 - 12x$, the values of x where the slope is zero.**

A $x = 3$ and $x = -3$

B $x = 1$ and $x = 0$

C $x = 1$ and $x = -1$

D $x = 2$ and $x = -2$

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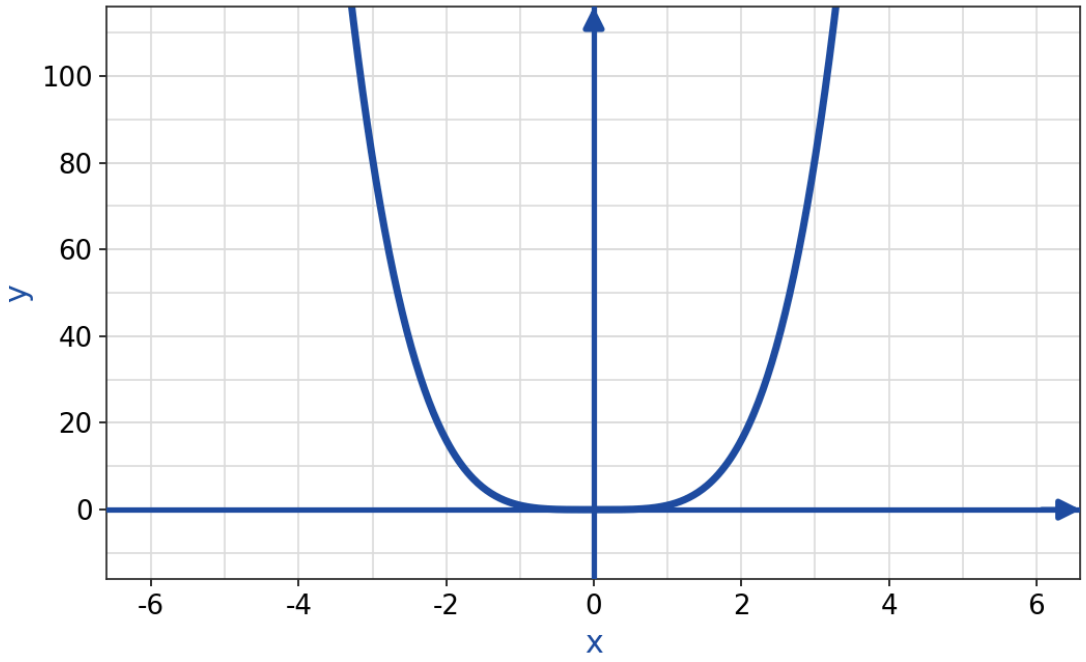
7 The curve $y = x^4$ flattens out at one point. Find the x value of that point.



A $x = 1$

B $x = -1$

C $x = 4$

D $x = 0$

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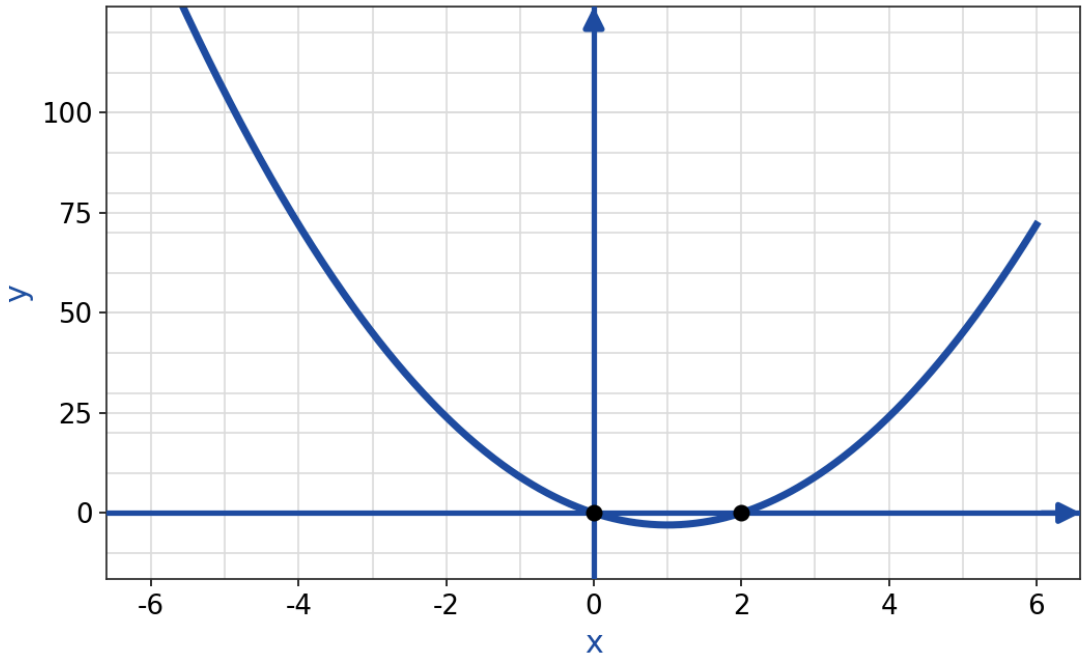
8 For $f(x) = 3x^2 - 6x$, find the value of x where the graph stops falling and starts rising.



A x = -1

B x = 1

C x = 0

D x = 3

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9 For $f(x) = x^2 + 1$, what is the lowest VALUE of the function?

- A 0
- B 1
- C 2
- D -1

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10 For $f(x) = 2x^3 - 6x$, at which critical point is the graph at its highest?

- A $x = 1$
- B $x = -1$
- C $x = 0$
- D $x = 2$

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ANSWER KEY

1. x = -1

2. x = 0 and x = 2

3. x = 0

4. x = 1 and x = 3

5. x = 2

6. x = 2 and x = -2

7. x = 0

8. x = 1

9. 1

10. x = -1

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