MATHEMATICS · GRADES 10-12

Derivative from first principles

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is the idea behind how a speedometer works out your speed right now. It looks at a tiny change in distance over a tiny change in time, so small that it is practically this instant. The same idea measures how fast anything is changing at one moment, like a temperature dropping or a balloon filling. It is the starting point for all of calculus, and it is how a single moment of change is measured.



Rule

Derivative from first principles: f'(x) is the limit of $\frac{f(x+h)-f(x)}{h}$, as h approaches 0.

It measures the rate of change at one instant by looking at a tiny change over a tiny interval and letting the interval shrink to nothing.

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Worked examples

EXAMPLE 1

Q **Find the derivative of $f(x) = x^2$ from first principles.**

SOLUTION

Write the difference quotient: $\frac{((x + h)^2 - x^2)}{h}$

Work out the top first: $(x + h)^2 - x^2 = 2xh + h^2$

Now divide each part by h: $2xh \div h = 2x$ and $h^2 \div h = h$, so the quotient is $2x + h$

Let h approach 0: the derivative is $2x$

EXAMPLE 2

Q **Find the derivative of $f(x) = 3x$ from first principles.**

SOLUTION

Difference quotient: work out the top first, $3(x + h) - 3x = 3h$, so the quotient is $\frac{3h}{h}$

Simplify: the h on the top cancels with the h on the bottom, leaving 3

Let h approach 0: the derivative is 3

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Practice questions

1 Write the definition of the derivative from first principles.

A Limit as h approaches 0 of $\frac{(f(x + h) - f(x))}{h}$

B Limit as h approaches 0 of $\frac{(f(x + h) + f(x))}{h}$

C Limit as h approaches infinity of $\frac{(f(x + h) - f(x))}{h}$

D $\frac{(f(x + h) - f(x))}{h}$, without taking a limit

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2 Find the derivative of $f(x) = x$ from first principles.

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3 Find the derivative of $f(x) = 5$ from first principles.

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4 Find the derivative of $f(x) = 2x$ from first principles.

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5 In the first-principles definition, what does h stand for?

- **A** A small change in y
- **B** A small change in x
- **C** The derivative of f
- **D** The slope of f at a point

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6 Find the derivative of $f(x) = 3x^2$ from first principles.

- **A** 6x
- **B** 3x
- **C** 6
- **D** $3x^2$

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7 Why does the definition let h approach 0?

- **A** To make h as large as possible
- **B** To find the average rate of change
- **C** To shrink the interval to a single point
- **D** To remove x from the formula

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8 **Find the derivative of $f(x) = 4x$ from first principles.**

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9 **Before you take the limit, what does $\frac{(f(x+h) - f(x))}{h}$ measure?**

A. The instantaneous rate of change at a point

B. The average rate of change over an interval of width h

C. The area under the curve

D. The value of f at x + h

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10 **Find the derivative of $f(x) = x^3$ from first principles.**

- **A** $3x^2$
- **B** x^3
- **C** $3x$
- **D** $6x$

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ANSWER KEY

1. Limit as h approaches 0 of

$$\frac{(f(x + h) - f(x))}{h}$$

2. 1

3. 0

4. 2

5. A small change in x

6. 6x

7. To shrink the interval to a single point

8. 4

9. The average rate of change over an interval of width h

10. $3x^2$

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