MATHEMATICS · GRADES 10-12

Derivatives of logarithmic functions

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you measure how fast something is changing when it is tracked on a scale like loudness or earthquake strength. On those scales each step up means a much bigger jump, so a small change in the number means a huge change in the real thing. The derivative tells you how fast the reading is climbing at any moment. It is how scientists track changes that would be impossible to read on a plain number line.

Rule

The derivative of ln x is $\frac{1}{x}$.

More generally, the derivative of ln of something is the derivative of that something divided by that something. So the derivative of ln(kx) is still $\frac{1}{x}$.

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Worked examples

EXAMPLE 1

Q Differentiate $\ln(5x)$.

SOLUTION

This is a log of something, so the thing to remember is: differentiate the inside, then divide by the inside.

The something is 5x, its derivative is 5, so the derivative is 5 over 5x

5 over 5x simplifies to 1 over x, so the answer is $\frac{1}{x}$. The trap here is stopping at $\frac{5}{5x}$ without simplifying.

EXAMPLE 2

Q Differentiate $\ln(x^2 + 1)$.

SOLUTION

Again, ln of something: derivative of the inside over the inside.

The inside is $x^2 + 1$, and its derivative is 2x

So the derivative is $\frac{2x}{x^2 + 1}$

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Practice questions

1 Differentiate $3 \ln x$.

- **A** $\frac{3}{x}$
- **B** $\frac{1}{x}$
- **C** $3x$
- **D** $\frac{x}{3}$

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2 Differentiate $\ln(7x)$.

- **A** $\frac{7}{x}$
- **B** $\frac{1}{x}$
- **C** $\frac{1}{7x}$
- **D** 7

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3 Differentiate $\ln(x^3)$.

- **A** $\frac{1}{x}$
- **B** $\frac{3}{x}$
- **C** $3x^2$
- **D** $\frac{x}{3}$

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4 Differentiate $\log_3 x$.

- **A** $\frac{1}{x \ln 3}$
- **B** $\frac{\ln 3}{x}$
- **C** $\frac{1}{\ln 3}$
- **D** $\frac{3}{x}$

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5 Differentiate $\ln(\cos x)$.

A $\tan x$

B $-\tan x$

C $-\cot x$

D $\sec x$

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6 Differentiate $\ln x + x^2$.

A $\frac{1}{x} + 2x$

B $\frac{1}{x} + x^2$

C $\frac{2}{x} + 2x$

D $\frac{1}{x} - 2x$

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7 Differentiate $x \ln x$.

A $\ln x$

B $\ln x + 1$

C $1 + \frac{1}{x}$

D $\ln x + x$

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8 Differentiate $\ln(e^x + 1)$.

A $\frac{e^x}{e^x + 1}$

B $\frac{1}{e^x + 1}$

C e^x

D $\frac{e^x + 1}{e^x}$

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9 Simplify $\ln(x^2)$ using a log law first, then differentiate.

A $\frac{2}{x}$

B $\frac{1}{x}$

C $2x$

D $\frac{x}{2}$

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10 Find $f'(1)$ for $f(x) = \ln(x^2 + 3)$.

A $\frac{1}{2}$

B 2

C $\frac{1}{4}$

D $\frac{1}{3}$

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ANSWER KEY

1. $\frac{3}{x}$

2. $\frac{1}{x}$

3. $\frac{3}{x}$

4. $\frac{1}{x \ln 3}$

5. $-\tan x$

6. $\frac{1}{x} + 2x$

7. $\ln x + 1$

8. $\frac{e^x}{e^x + 1}$

9. $\frac{2}{x}$

10. $\frac{1}{2}$

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