MATHEMATICS · GRADES 10-12

Factoring using substitution

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you swap a messy chunk of a formula for a single letter, the way you would use a nickname for a long address. Once the messy part has a short name, the pattern underneath becomes clear and the factoring is easy. You swap the short name back at the end to get the real answer. It is a way of seeing the forest instead of getting lost in the trees.



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Worked examples

EXAMPLE 1

Q Factor $x^4 + 5x^2 + 6$

SOLUTION

Let $u = x^2$, so x^4 becomes u^2 and the expression becomes $u^2 + 5u + 6$

Factor in u: $(u + 2)(u + 3)$

Put x^2 back for u: $(x^2 + 2)(x^2 + 3)$

EXAMPLE 2

Q Factor $(x + 1)^2 + 3(x + 1) + 2$

SOLUTION

Let $u = x + 1$, so the expression becomes $u^2 + 3u + 2$

Factor in u: $(u + 1)(u + 2)$

Put $x + 1$ back for u: $(x + 2)(x + 3)$

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Practice questions

1 Factor $x^4 + 7x^2 + 12$

- **A** $(x^2 + 3)(x^2 + 4)$
- **B** $(x^2 - 3)(x^2 - 4)$
- **C** $(x^2 + 3)(x^2 - 4)$
- **D** $(x^2 - 3)(x^2 + 4)$

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2 Factor $x^4 - 5x^2 + 4$

- **A** $(x^2 + 1)(x^2 + 4)$
- **B** $(x^2 - 1)(x^2 + 4)$
- **C** $(x^2 + 1)(x^2 - 4)$
- **D** $(x^2 - 1)(x^2 - 4)$

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3 Factor $(x + 2)^2 + 5(x + 2) + 6$

A $(x + 2)(x + 3)$

B $(x + 4)(x + 5)$

C $(x + 5)(x + 6)$

D $(x + 4)(x + 6)$

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4 Factor $x^4 + 3x^2 - 10$

A $(x^2 - 5)(x^2 + 2)$

B $(x^2 + 5)(x^2 + 2)$

C $(x^2 + 5)(x^2 - 2)$

D $(x^2 - 5)(x^2 - 2)$

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5 Factor $(2x - 1)^2 - 4(2x - 1) + 3$

A $4(x + 1)(x + 2)$

B $4(x - 1)(x - 2)$

C $(x - 1)(x - 2)$

D $4(x - 1)(x + 2)$

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6 Factor $x^4 + 6x^2 + 9$

A $(x^2 - 3)^2$

B $(x^2 + 3)(x^2 - 3)$

C $(x^2 + 6)(x^2 + 3)$

D $(x^2 + 3)^2$

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7 **The factored form is $(x^2 + 6)(x^2 + 1)$. What was the original expression?**

- **A** $x^4 + 7x^2 + 6$
- **B** $x^4 + 6x^2 + 6$
- **C** $x^4 + 7x^2 + 7$
- **D** $x^4 + 5x^2 + 6$

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8 **Why does letting $u = x^2$ turn $x^4 + 5x^2 + 6$ into a quadratic you already know how to factor?**

- **A** x^4 is $(x^2)^2$, so with $u = x^2$ it becomes $u^2 + 5u + 6$, which factors as $(u + 2)(u + 3)$
- **B** $u = x^2$ makes the x^4 term disappear
- **C** Substituting $u = x^2$ turns every x into u, giving $u^4 + 5u^2 + 6$
- **D** x^4 is x times x times x times x, so u must be x^4

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9 **Factor $x^4 + 8x^2 + 7$**

- **A** $(x^2 + 1)(x^2 + 7)$
- **B** $(x^2 + 1)(x^2 + 6)$
- **C** $(x^2 + 1)(x^2 + 5)$
- **D** $(x^2 + 1)(x^2 + 4)$

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10 **Which one does not factor with whole numbers?**

A $x^4 + 5x^2 + 6$

B $x^4 + 6x^2 + 8$

C $x^4 + 7x^2 + 10$

D $x^4 + 5x^2 + 7$

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ANSWER KEY

1. $(x^2 + 3)(x^2 + 4)$

2. $(x^2 - 1)(x^2 - 4)$

3. $(x + 4)(x + 5)$

4. $(x^2 + 5)(x^2 - 2)$

5. $4(x - 1)(x - 2)$

6. $(x^2 + 3)^2$

7. $x^4 + 7x^2 + 6$

8. x^4 is $(x^2)^2$, so with $u = x^2$ it becomes $u^2 + 5u + 6$, which factors as $(u + 2)(u + 3)$

9. $(x^2 + 1)(x^2 + 7)$

10. $x^4 + 5x^2 + 7$

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