MATHEMATICS · GRADES 10-12

Graphing rational functions

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you picture a relationship with a limit it can never cross, like the average cost per item as you make more and more of them. The curve keeps getting closer to a certain value without ever reaching it, and the graph shows that limit clearly. You see it in the average cost of making more products, the concentration of a solution as you keep adding water, and the time it takes to finish a job as more people help.



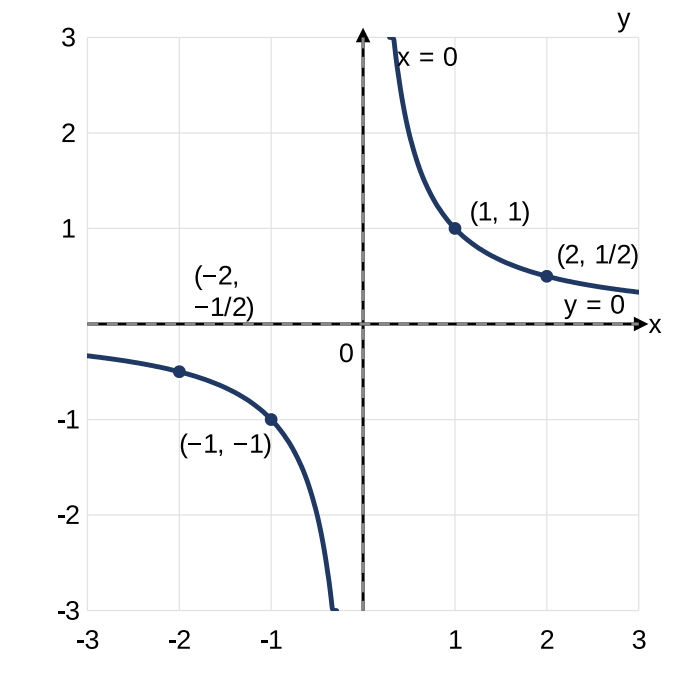
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Worked examples

EXAMPLE 1

Q Graph $f(x) = \frac{1}{x}$

SOLUTION



Asymptotes: vertical $x = 0$ and horizontal $y = 0$

Points: $(1, 1), (2, \frac{1}{2}), (-1, -1), (-2, -\frac{1}{2})$

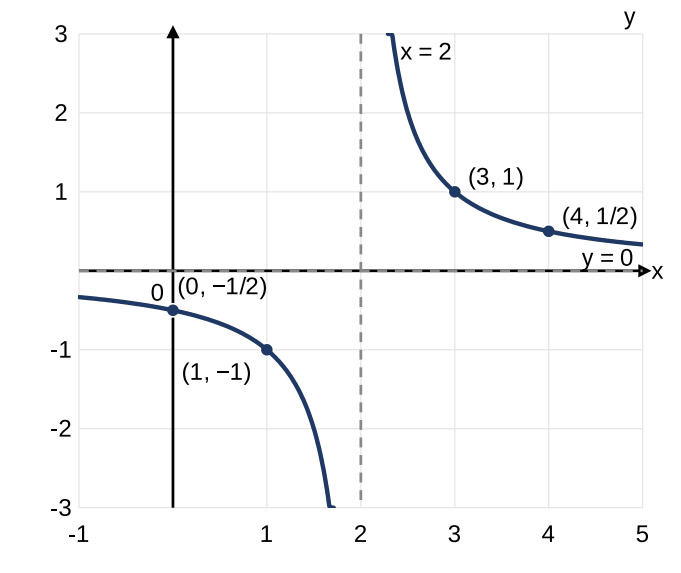
Draw two branches, one in the first quadrant and one in the third, approaching but never touching the axes.

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EXAMPLE 2

Q Graph $f(x) = \dfrac{1}{(x - 2)}$

SOLUTION



Asymptotes: vertical $x = 2$ and horizontal $y = 0$

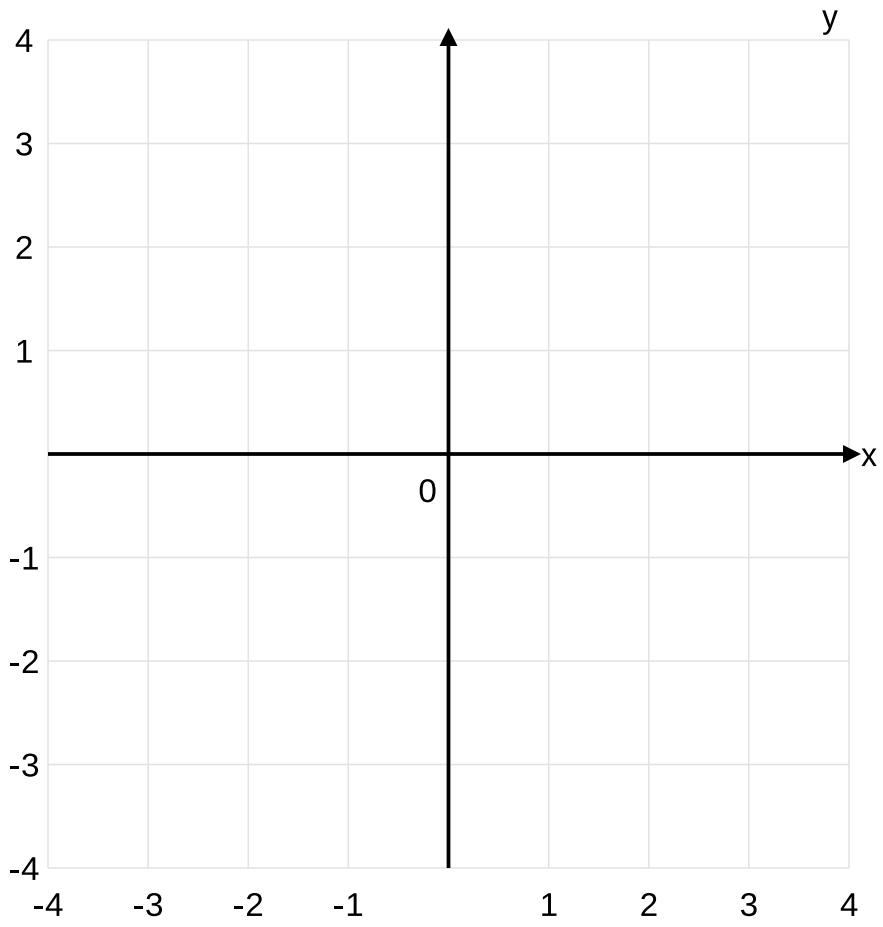
Points: $(3, 1)$, $(4, \frac{1}{2})$, $(1, -1)$, $(0, -\frac{1}{2})$

Draw two branches approaching $x = 2$ and $y = 0$, shifted right by 2 from the graph of $\frac{1}{x}$.

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Practice questions

1 Graph $f(x) = \frac{-1}{x}$, giving the points you use and both asymptotes.



A Points $(1, -1)$, $(2, -\frac{1}{2})$, $(-1, 1)$, $(-2, \frac{1}{2})$; vertical asymptote $x = 0$, horizontal asymptote $y = 0$; two branches in the second and fourth quadrants

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B

Points $(3, 1), (4, \frac{1}{2}), (1, -1), (0, -\frac{1}{2})$; vertical asymptote $x = 2$, horizontal asymptote $y = 0$; two branches shifted right by 2

C

Points $(1, -1), (2, -\frac{1}{2}), (-1, 1), (-2, \frac{1}{2})$; vertical asymptote $x = 0$, horizontal asymptote $y = 0$; two branches in the first and third quadrants

D

Points $(1, 1), (2, \frac{1}{2}), (-1, -1), (-2, -\frac{1}{2})$; vertical asymptote $x = 0$, horizontal asymptote $y = 0$; two branches in the second and fourth quadrants

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2 **Graph $f(x) = \frac{1}{(x + 2)}$, giving the points you use and both asymptotes.**

A Points (1, 1), (2, $\frac{1}{2}$), (-1, -1), (-2, -$\frac{1}{2}$); vertical asymptote x = 0, horizontal asymptote y = 0; two branches in the first and third quadrants

B Points (-1, 1), (0, $\frac{1}{2}$), (-3, -1), (-4, -$\frac{1}{2}$); vertical asymptote x = -2, horizontal asymptote y = 0; two branches shifted left by 2

C Points (-1, 1), (0, $\frac{1}{2}$), (-3, -1), (-4, -$\frac{1}{2}$); vertical asymptote x = 2, horizontal asymptote y = 0; two branches shifted right by 2

D Points (-1, 1), (0, $\frac{1}{2}$), (-3, -1), (-4, -$\frac{1}{2}$); vertical asymptote x = -2, horizontal asymptote y = 0; two branches shifted right by 2

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3 **What is the domain of $f(x) = \frac{1}{x+1}$?**

A $x \neq -1$

B $x \neq 1$

C $x > 0$

D All real numbers

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4 **Graph $f(x) = \frac{2}{x}$, giving the points you use and both asymptotes.**

A Points (1, 1), (2, $\frac{1}{2}$), (-1, -1), (-2, $-\frac{1}{2}$); vertical asymptote x = 0, horizontal asymptote y = 0; two branches in the first and third quadrants

B Points (1, 2), (2, 1), (-1, -2), (-2, -1); vertical asymptote x = 0, horizontal asymptote y = 0; two branches in the second and fourth quadrants

C Points (1, 2), (2, 1), (-1, -2), (-2, -1); vertical asymptote x = 0, horizontal asymptote y = 0; two branches in the first and third quadrants

D Points (1, 1), (2, $\frac{1}{2}$), (-1, -1), (-2, $-\frac{1}{2}$); vertical asymptote x = 0, horizontal asymptote y = 1; two branches in the first and third quadrants

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5 Find $f(4)$ for $f(x) = \frac{1}{x}$.

A $\frac{1}{4}$

B 4

C $\frac{1}{2}$

D $-\frac{1}{4}$

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6 For $f(x) = \frac{1}{x}$, find x when $f(x) = 2$.

A $\frac{1}{2}$

B 2

C -2

D $-\frac{1}{2}$

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7 State the vertical asymptote of $f(x) = \frac{1}{(x - 1)}$.

- **A** x = 1
- **B** x = -1
- **C** y = 1
- **D** x = 0

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8 What is the horizontal asymptote of $f(x) = \frac{1}{x + 3}$?

- **A** y = 0
- **B** y = -3
- **C** x = -3
- **D** y = 3

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9 As x gets very large, the graph of $f(x) = \frac{1}{x}$ levels off toward which line?

- **A** y = 1
- **B** y = -1
- **C** y = 0
- **D** y = 2

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10 Find $f(3)$ for $f(x) = \frac{3}{x}$.

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ANSWER KEY

1. Points $(1, -1)$, $(2, -\frac{1}{2})$, $(-1, 1)$, $(-2, \frac{1}{2})$; vertical asymptote $x = 0$, horizontal asymptote $y = 0$; two branches in the second and fourth quadrants

2. Points $(-1, 1)$, $(0, \frac{1}{2})$, $(-3, -1)$, $(-4, -\frac{1}{2})$; vertical asymptote $x = -2$, horizontal asymptote $y = 0$; two branches shifted left by 2

3. $x \neq -1$

4. Points $(1, 2)$, $(2, 1)$, $(-1, -2)$, $(-2, -1)$; vertical asymptote $x = 0$, horizontal asymptote $y = 0$; two branches in the first and third quadrants

5. $\frac{1}{4}$

6. $\frac{1}{2}$

7. $x = 1$

8. $y = 0$

9. $y = 0$

10. 1

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