MATHEMATICS · GRADES 10-12

Implicit differentiation

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you find a rate of change when two things are tangled together in one equation, like how fast a shadow grows as a person walks away from a lamp. Some relationships cannot be untangled into a simple formula first, so you have to work with the equation as it is. It shows up in real problems like a ladder sliding down a wall, a balloon inflating, or a shadow lengthening, where two quantities change together and you need the rate of one from the rate of the other.



Rule

When you differentiate an equation that has y in it, differentiate every y term with the chain rule, so each one picks up a y'.

Then solve the result for y', the same way you solve any equation for an unknown.

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Worked examples

EXAMPLE 1

Q Find y' for $x^2 + y^2 = 25$.

SOLUTION

Differentiate term by term: $2x + 2y \times y' = 0$

The y term needs the chain rule, which is where y' comes from.

Solve: $y' = -\frac{x}{y}$

EXAMPLE 2

Q Find y' for $x \times y = 6$.

SOLUTION

Use the product rule on $x \times y$: $y + x \times y' = 0$

So $x \times y' = -y$

So $y' = -\frac{y}{x}$

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Practice questions

1 Find y' for $x^2 + y^2 = 49$.

A $\frac{-x}{y}$

B $\frac{x}{y}$

C $\frac{-y}{x}$

D $\frac{y}{x}$

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2 Find y' for $x \times y = 12$.

A $\frac{-x}{y}$

B $\frac{-y}{x}$

C $\frac{y}{x}$

D $\frac{x}{y}$

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3 Find y' for $x + y = 10$.

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4 Find y' for $x^2 + y = 9$.

- **A** $-2x$
- **B** $2x$
- **C** $-x$
- **D** 2

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5 Find y' for $y^2 = x$.

- **A** $\frac{1}{2y}$
- **B** $2y$
- **C** $\frac{1}{y}$
- **D** $\frac{2}{y}$

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6 Find y' for $x^3 + y^3 = 8$.

A $\frac{-x^2}{y^2}$

B $\frac{x^2}{y^2}$

C $\frac{-y^2}{x^2}$

D $\frac{-x}{y}$

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7 Find y' for $x^2 - y^2 = 1$.

A $\frac{x}{y}$

B $\frac{-x}{y}$

C $\frac{y}{x}$

D $\frac{-y}{x}$

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8 Find y' for $2x + 3y = 6$.

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9 Find y' for $y^2 = 4x$.

A $\frac{2}{y}$

B 2

C $\frac{y}{2}$

D $\frac{4}{y}$

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10

Find y' for $x^3 + y = 10$.

A $-3x^2$

B $3x^2$

C $-x^3$

D $-3x$

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ANSWER KEY

1. $\frac{-x}{y}$

2. $\frac{-y}{x}$

3. -1

4. $-2x$

5. $\frac{1}{2y}$

6. $\frac{-x^2}{y^2}$

7. $\frac{x}{y}$

8. $\frac{-2}{3}$

9. $\frac{2}{y}$

10. $-3x^2$

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