MATHEMATICS · GRADES 10-12

Infinite geometric series

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you add up a pattern that shrinks forever, like the total distance a bouncing ball travels as each bounce gets smaller and smaller. The bounces never quite stop, but they get so small that the total distance settles on one finite number, which is the surprising part. You meet the same idea in a loan that is paid off in shrinking pieces, in a signal that fades away, or in any process that keeps halving until there is almost nothing left to add.



Rule

An infinite geometric series has a finite sum only when the size of the common ratio is less than 1.

When it is, the sum is $\frac{a}{(1 - r)}$, where a is the first term and r is the common ratio.

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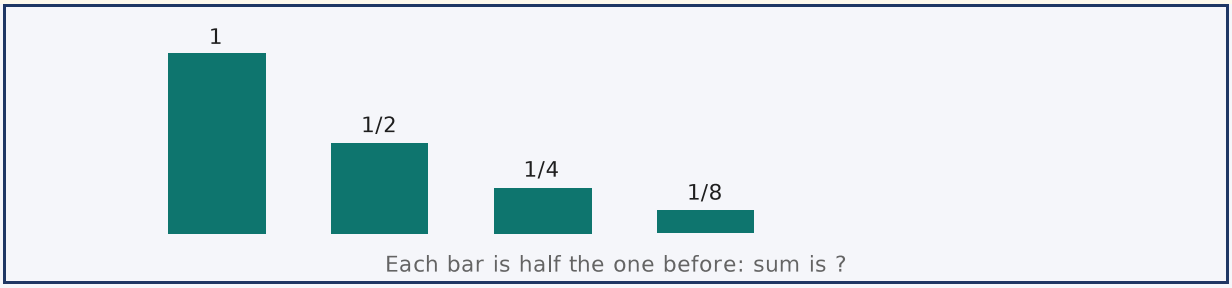
Worked examples

EXAMPLE 1

Q Find $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ and so on, forever.



SOLUTION

Here a = 1 and r = $\frac{1}{2}$, and the size of r is less than 1, so the sum settles.

Formula: Sum = $\frac{a}{(1 - r)}$

Put in a = 1 and r = $\frac{1}{2}$:

Sum = $\frac{1}{(1 - \frac{1}{2})} = \frac{1}{\frac{1}{2}} = 2$

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EXAMPLE 2

Q Find $9 + 3 + 1 + \frac{1}{3}$ and so on, forever.

SOLUTION

Here $a = 9$ and $r = \frac{3}{9} = \frac{1}{3}$.

$$\text{Sum} = \frac{9}{(1 - \frac{1}{3})} = \frac{9}{\frac{2}{3}}$$

Sum = 13.5

Practice questions

1 A ball bounces 1 m, then $\frac{1}{3}$ m, then $\frac{1}{9}$ m, and so on, forever. How far does it travel in total?

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2 An infinite geometric series has first term 8 and sum 16. What is the common ratio?

- **A** $\frac{1}{2}$
- **B** 2
- **C** $\frac{1}{4}$
- **D** $\frac{1}{3}$

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3 What is the 5th term of the series 10, 1, 0.1, and so on?

- **A** 0.001
- **B** 0.01
- **C** 0.0001
- **D** 0.1

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What decides whether an infinite geometric series has a sum?

A The first term is less than 1

B The size of the common ratio is less than 1

C The common ratio is greater than 1

D The terms keep getting bigger

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5

A rumour doubles each day. Does the total ever settle to a finite sum?

A Yes

B No

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6 What is the sum of the first 2 terms of $6 + 2 + \frac{2}{3} + \ldots$?

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7 Which of these series has a sum?

A. $12 + 6 + 3 + \ldots$

B. $12 + 24 + 48 + \ldots$

C. $12 + 12 + 12 + \ldots$

D. $12 + 36 + 108 + \ldots$

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8 The sum of $5 + 4 + 3.2 + \ldots$ is 25. What would the sum be if the first term doubled to 10?

- **A** 50
- **B** 25
- **C** 20
- **D** 100

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9 Write the formula for the sum of an infinite geometric series with first term a and common ratio r.

- **A** $\text{Sum} = \frac{a}{(1 - r)}$
- **B** $\text{Sum} = \frac{a}{(1 + r)}$
- **C** $\text{Sum} = a(1 - r)$
- **D** $\text{Sum} = a + r$

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10 **A pond is filled with 2 L, then 1 L, then 0.5 L, and so on. After how many fills has 3.5 L been added?**

A 2

B 3

C 4

D It never reaches 3.5 L

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ANSWER KEY

1. 1.5

2. $\frac{1}{2}$

3. 0.001

4. The size of the common ratio is less than 1

5. No

6. 8

7. 12 + 6 + 3 + ...

8. 50

9. Sum = $\frac{a}{(1 - r)}$

10. 3

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