MATHEMATICS · GRADES 10-12

Natural logarithms

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you work out how long money takes to double in a savings account, how fast a population grows, or how quickly something cools down. A bank uses it to compare investment options, and a scientist uses it to work out how long a medicine stays in the body or how fast bacteria multiply. Anything that grows or shrinks by a percentage over time follows this same pattern. Once you can read that pattern, you can predict what happens next instead of just guessing.



Rule

A natural logarithm has base e, so ln(x) asks: to what power must e be raised to get x?

That means $\ln(e^n) = n$, and $\ln(1) = 0$.

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Worked examples

EXAMPLE 1

Q Solve $e^x = 20$.

SOLUTION

Take the natural log of both sides. ln undoes e, so the left side becomes x on its own.

x = ln 20 ≈ 3

EXAMPLE 2

Q How long does $1000 take to double at 5% interest, compounded continuously?

SOLUTION

Doubling means the amount is twice what it started at, so e^(0.05t) = 2. Take ln of both sides.

0.05t = ln 2 ≈ 0.69

t = 0.69 ÷ 0.05 ≈ 13.9

About 13.9 years.

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Practice questions

1 Write $e^4 = 54.6$ in logarithmic form.

- **A** $\ln 54.6 = 4$
- **B** $\ln 4 = 54.6$
- **C** $e^4 = \ln 54.6$
- **D** $54.6 = e^4$

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2 What is $\ln(e^7)$?

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What is ln 1?

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Is ln 0.5 positive or negative?

A Positive

B Negative

C Zero

D Undefined

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5 For which x is ln x undefined?

- **A** x = 2
- **B** x = 0
- **C** x = 5
- **D** x = 1

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6 Solve $e^x = 10$. Give your answer as a natural log, then about how big it is.

- **A** ln 10, about 2.3
- **B** ln 10, about 3.0
- **C** ln 2, about 0.7
- **D** ln 5, about 1.6

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7 Solve $3e^x = 15$.

- **A** ln 3, about 1.1
- **B** ln 15, about 2.7
- **C** ln 5, about 1.6
- **D** ln 5, about 2.3

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8 A population doubles every 8 years and grows as $P = Ae^{kt}$. What is k?

- **A** About 0.087
- **B** About 0.125
- **C** About 0.25
- **D** About 0.5

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9 $500 grows to $1000 at 4% interest, compounded continuously. How long does it take?

- **A** About 10 years
- **B** About 17.3 years
- **C** About 25 years
- **D** About 34.7 years

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10 What happens to ln x as x gets closer and closer to 0?

- **A** It goes to $-\infty$
- **B** It goes to ∞
- **C** It stays at 0
- **D** It goes to 1

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ANSWER KEY

1. $\ln 54.6 = 4$

2. 7

3. 0

4. Negative

5. $x = 0$

6. $\ln 10$, about 2.3

7. $\ln 5$, about 1.6

8. About 0.087

9. About 17.3 years

10. It goes to $-\infty$

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