MATHEMATICS · GRADES 10-12

Number and nature of roots

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you know before you start solving whether a problem like finding when a ball hits the ground has two answers, one answer, or no answer at all. A ball thrown in the air crosses the same height twice, once on the way up and once on the way down, and that tells you to expect two answers. A business working out its break-even point asks the same question, since it might break even twice, once, or never. Knowing how many answers to expect keeps you from hunting for one that does not exist, or stopping after the first when there is a second.



Rule

The discriminant is $D = b^2 - 4ac$. It tells you the roots before you solve.

If it is positive, there are two distinct real roots. If it is zero, there is one repeated real root. If it is negative, there are no real roots.

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Worked examples

EXAMPLE 1

Q Describe the roots of $x^2 - 6x + 8 = 0$

SOLUTION

First read off a, b and c from the equation: a = 1, b = -6 and c = 8.

The discriminant is $D = b^2 - 4ac$. Here $b^2 = (-6)^2 = 36$, and $4ac = 4 \times 1 \times 8 = 32$, so $D = 36 - 32 = 4$.

The discriminant is positive, so the equation has two distinct real roots.

Solving gives x = 2 or x = 4, which confirms it.

EXAMPLE 2

Q Describe the roots of $x^2 + 2x + 5 = 0$

SOLUTION

First read off a, b and c from the equation: a = 1, b = 2 and c = 5.

The discriminant is $D = b^2 - 4ac$. Here $b^2 = 2^2 = 4$, and $4ac = 4 \times 1 \times 5 = 20$, so $D = 4 - 20 = -16$.

The discriminant is negative, so the equation has no real roots at all.

The roots are a pair of complex conjugates (two matching numbers that are not real).

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Practice questions

1 Does $x^2 + 5x + 6 = 0$ have two distinct real roots, one repeated real root, or no real roots?

- **A** two distinct real roots
- **B** one repeated real root
- **C** no real roots

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2 How many real roots does $x^2 + 4x + 4 = 0$ have, and are they different?

- **A** two distinct real roots
- **B** one repeated real root
- **C** no real roots

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3 Does $x^2 + x + 3 = 0$ have two distinct real roots, one repeated real root, or no real roots?

- A two distinct real roots
- B one repeated real root
- C no real roots

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4 What is the value of the discriminant of $x^2 - 9x + 20 = 0$?

- A 1
- B -1
- C 161

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5

The discriminant of a quadratic equation is 36. How many real roots does the equation have?

A two distinct real roots

B one repeated real root

C no real roots

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6

A quadratic equation has a discriminant of -3. What does its graph do?

A It crosses the x-axis twice

B It touches the x-axis once

C It does not touch or cross the x-axis

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7 Only one of these equations has a single repeated real root. Which one?

A $x^2 + 4x + 4 = 0$

B $x^2 + 5x + 6 = 0$

C $x^2 + x + 1 = 0$

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8 Why does a discriminant of -7 mean there are no real roots?

A The discriminant is under the square root in the quadratic formula, and you cannot take the square root of a negative number and stay real

B The discriminant is the answer to the equation

C A negative discriminant means one repeated root

D The discriminant is always positive

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9 Which of these equations has no real roots?

- **A** $x^2 + 3x + 2 = 0$
- **B** $x^2 + 2x + 1 = 0$
- **C** $x^2 + x + 2 = 0$

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10 Only one of these equations has two distinct real roots. Which one?

- **A** $x^2 + 4x + 4 = 0$
- **B** $x^2 + x + 1 = 0$
- **C** $x^2 + 5x + 6 = 0$
- **D** $x^2 - 10x + 25 = 0$

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ANSWER KEY

1. two distinct real roots

2. one repeated real root

3. no real roots

4. 1

5. two distinct real roots

6. It does not touch or cross the x-axis

7. $x^2 + 4x + 4 = 0$

8. The discriminant is under the square root in the quadratic formula, and you cannot take the square root of a negative number and stay real

9. $x^2 + x + 2 = 0$

10. $x^2 + 5x + 6 = 0$

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