MATHEMATICS · GRADES 10-12

One-sided limits

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you work out what something is approaching from one side only, like the fare just below an age cutoff or a reading just before a gauge hits its limit. A child who is almost twelve pays the child fare right up to the birthday, then the price jumps, so the value just below the cutoff matters. A phone plan works the same way, with one rate up to a data cap and a different rate after it. Looking from one side only tells you what happens just before the change, which is often the number you actually need.



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Rule

A limit exists only if the left-hand limit and the right-hand limit are the same.

If the two sides agree, that shared value is the limit. If they disagree, the limit does not exist.

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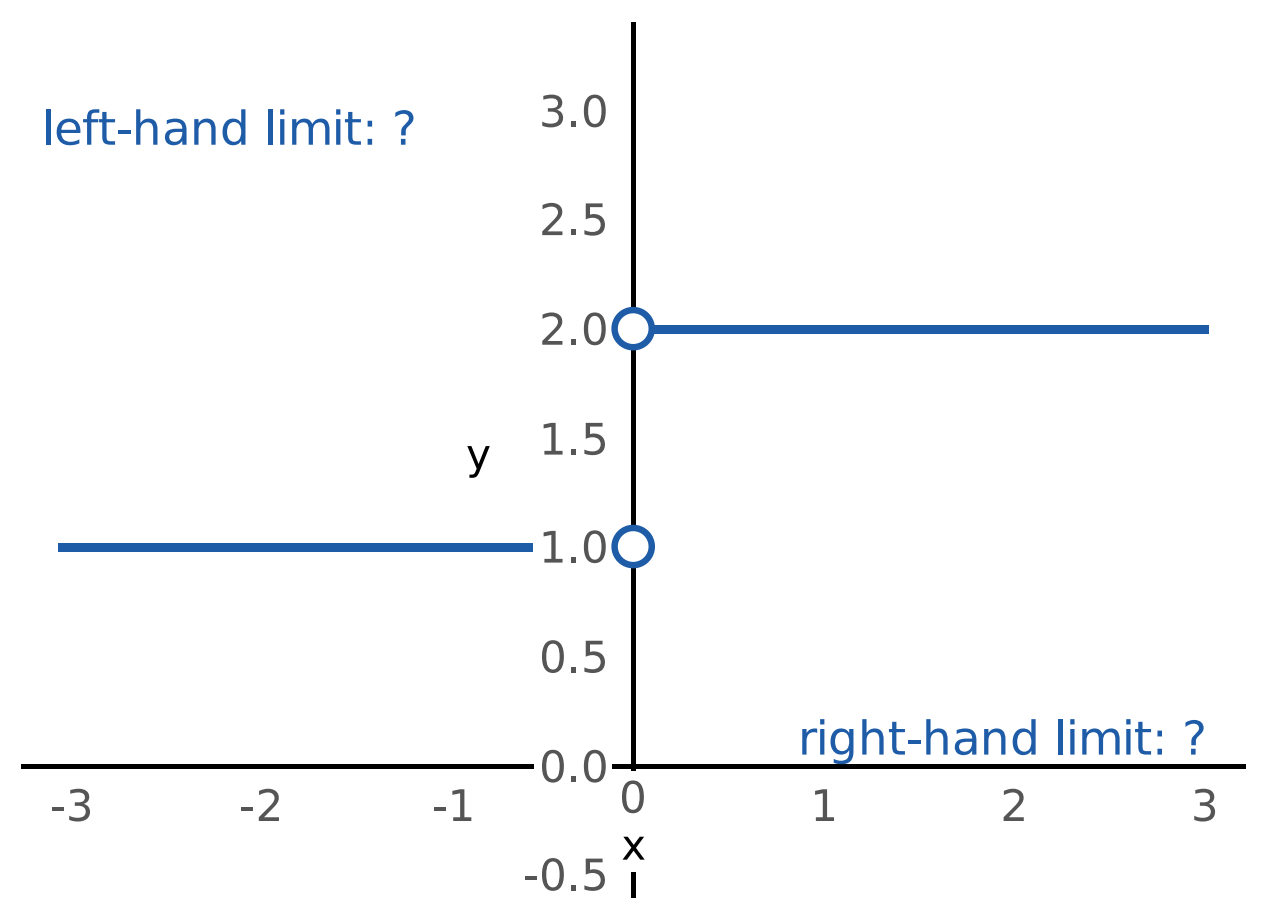
Worked examples

EXAMPLE 1

Q For f(x) = 1 when x is less than 0, and 2 when x is greater than 0, find the left and right limits at 0.



SOLUTION

From the left, x is negative, so f(x) = 1: left limit 1.

From the right, x is positive, so f(x) = 2: right limit 2.

The two sides disagree, so the ordinary limit at 0 does not exist.

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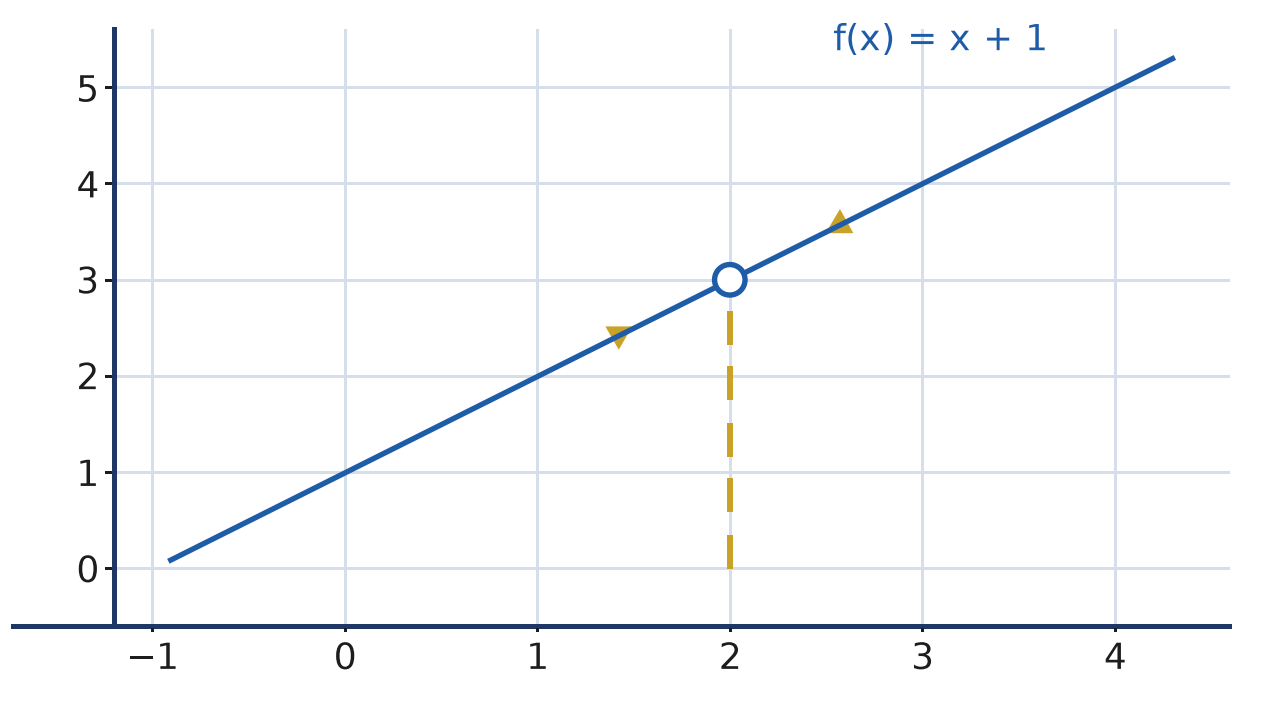
EXAMPLE 2

Q For f(x) = x + 1 everywhere, find both one-sided limits at 2.



SOLUTION

From the left, x gets close to 2, so x + 1 gets close to 2 + 1 = 3.

From the right, x also gets close to 2, so x + 1 also gets close to 3.

Both sides agree, so the limit at 2 is 3.

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Practice questions

1 **A child fare is $1 for ages under 12 and $2 for ages over 12. Find the left and right limits at 12. Does the ordinary limit exist?**

- **A** Left limit 1, right limit 2; the ordinary limit does not exist
- **B** Left limit 2, right limit 1; the ordinary limit does not exist
- **C** Both 1, so the limit at 12 is 1
- **D** Both 2, so the limit at 12 is 2

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2 For f(x) = x + 1 everywhere, find both one-sided limits at 5.

- **A** Both 5, so the limit at 5 is 5
- **B** Both 6, so the limit at 5 is 6
- **C** Left limit 5, right limit 6; the ordinary limit does not exist
- **D** Both 1, so the limit at 5 is 1

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3 A phone plan charges $0 below the data cap and $1 above it. What is the left-hand limit at the cap?

- **A** 0
- **B** 1
- **C** The limit does not exist
- **D** Both 0 and 1

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4 For f(x) = 2x everywhere, what is the left-hand limit at 4?

- **A** 8
- **B** 4
- **C** 2
- **D** The limit does not exist

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5 For f(x) = 4 when x is less than 2, and 7 when x is greater than 2, what is the right-hand limit at 2?

- **A** 4
- **B** 7
- **C** The limit does not exist
- **D** Both 4 and 7

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6 For $f(x) = x^2$ everywhere, what is the right-hand limit at 2?

- **A** 2
- **B** 4
- **C** 16
- **D** The limit does not exist

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7 For $f(x) = x$ when x is less than 0, and $x + 1$ when x is greater than 0, find the left and right limits at 0. Does the ordinary limit exist?

- **A** Left limit 0, right limit 1; the ordinary limit does not exist
- **B** Left limit 1, right limit 0; the ordinary limit does not exist
- **C** Both 0, so the limit at 0 is 0
- **D** Both 1, so the limit at 0 is 1

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8 A function has left limit 4 and right limit 4 at $x = 1$, but $f(1) = 9$. What is the limit at 1?

- A 9
- B 4
- C The limit does not exist
- D 5

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9 If the left limit is 7 and the right limit is 7, what is the limit?

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10

If the left limit is 3 and the right limit is 4, does the limit exist?

A Yes

B No

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ANSWER KEY

1. Left limit 1, right limit 2; the ordinary limit does not exist

2. Both 6, so the limit at 5 is 6

3. 0

4. 8

5. 7

6. 4

7. Left limit 0, right limit 1; the ordinary limit does not exist

8. 4

9. 7

10. No

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