MATHEMATICS · GRADES 10-12

Optimization problems

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how a farmer gets the biggest field from a fixed length of fence, or a business finds the price that brings in the most money. A farmer with a set amount of fencing has to choose the shape that gives the most grazing land, and a shop has to find the price where higher sales and higher profit meet. A packaging company asks the same question when it wants the smallest box that still holds the product. Whenever there is a limit on one side and a best result on the other, this is the maths that finds the sweet spot.



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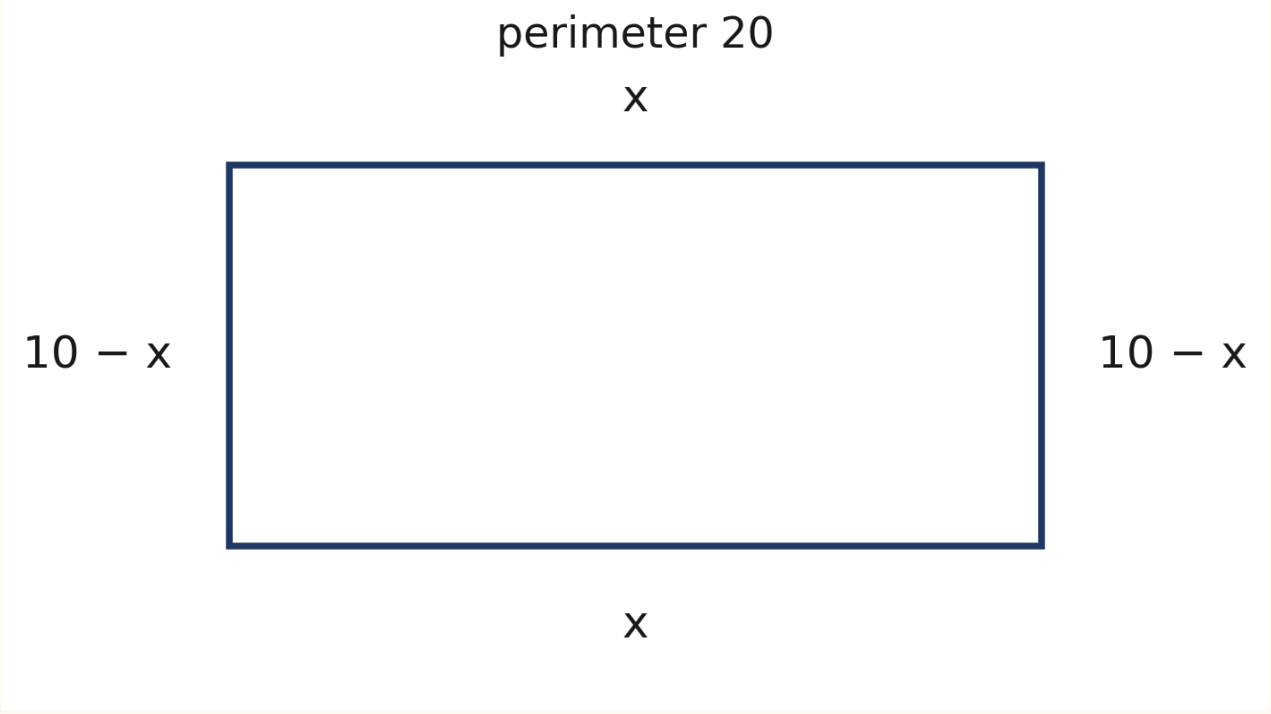
Worked examples

EXAMPLE 1

Q A rectangle has perimeter 20. What dimensions give the largest area?



SOLUTION

Let the sides be x and 10 - x, since 2x + 2y = 20 gives y = 10 - x.

Area = $x(10 - x) = 10x - x^2$

The area is largest where its slope is zero, so differentiate the area: the derivative of $10x - x^2$ is 10 - 2x.

Set it to 0: 10 - 2x = 0, so x = 5, and y = 5. A 5 by 5 square gives the largest area, 25

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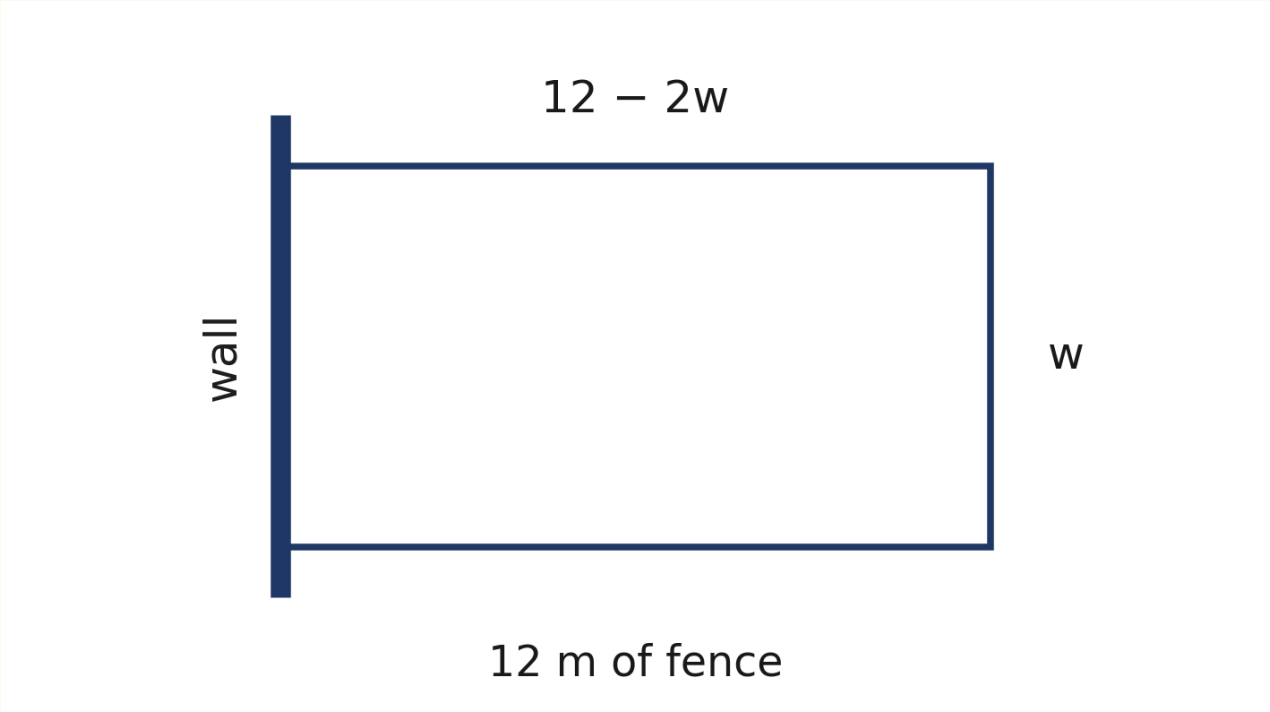
EXAMPLE 2

Q **You have 12 metres of fence for a rectangle against a wall, with only three sides fenced. What width gives the largest area?**



SOLUTION

Let the width be w and the length be 12 - 2w. Area = w(12 - 2w) = 12w - $2w^2$

The area is largest where its slope is zero, so differentiate the area: the derivative of 12w - $2w^2$ is 12 - 4w.

Set it to 0: 12 - 4w = 0, so w = 3. Then length = 6, area = 18 m^2.

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Practice questions

1. A rectangle has perimeter 12. What square side gives the largest area?

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2. A rectangle has perimeter 40. What is the largest area?

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3. You have 20 metres of fence for three sides against a wall. What width gives the largest area?

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4 You have 16 metres of fence for three sides against a wall. What length gives the largest area?

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5 You have 24 metres of fence for three sides against a wall. What width gives the largest area, and what is that area?

A. Width 4 metres, area 64 m^2

B. Width 8 metres, area 64 m^2

C. Width 3 metres, area 54 m^2

D. Width 6 metres, area 72 m^2

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6 **A rectangle has fixed area 36. What square side gives the smallest perimeter?**

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7 **A rectangle has fixed area 25. What is the smallest possible perimeter?**

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8 A rectangle has perimeter 30. What are the dimensions of largest area?

- **A** 7 by 8
- **B** 7.5 by 7.5
- **C** 6 by 9
- **D** 15 by 15

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9 You have 28 metres of fence for three sides against a wall. What is the largest area?

- **A** 98 m^2
- **B** 49 m^2
- **C** 196 m^2
- **D** 56 m^2

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10 You have 28 metres of fence for a rectangle against a wall, with only three sides fenced. What dimensions give the largest area?

- **A** 7 by 14
- **B** 14 by 14
- **C** 7 by 7
- **D** 28 by 14

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ANSWER KEY

1. 3

2. 100

3. 5 metres

4. 8 metres

5. Width 6 metres, area 72 m^2

6. 6

7. 20

8. 7.5 by 7.5

9. 98 m^2

10. 7 by 14

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