MATHEMATICS · GRADES 10-12

Phase shifts and vertical shifts

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you describe a wave that starts later or sits higher, like a tide that peaks an hour later each day or a temperature graph that sits higher in summer than in winter. A sailor plans around a tide that arrives later each day, and a gardener plans around temperatures that run higher all summer. Daylight hours shift through the year in the same way, with the whole pattern sliding earlier or later. Naming how a wave shifts lets you describe the change without redrawing the whole graph.



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Rule

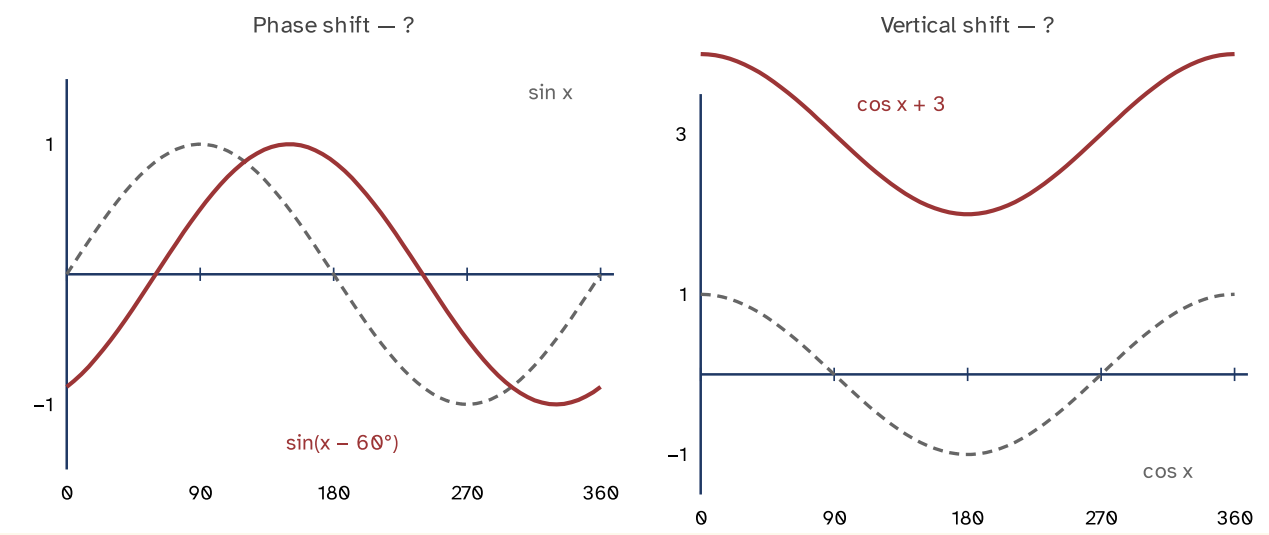
A number added or subtracted inside the brackets shifts the graph sideways, in the opposite direction: minus moves it right and plus moves it left.

A number added or subtracted outside shifts the graph up or down: plus moves it up and minus moves it down.

Worked examples

EXAMPLE 1

Q **Describe the shift in y = sin(x – 60°).**



SOLUTION

What is inside the brackets moves the graph sideways, and the direction is the opposite of the sign. x – 60° means the graph moves 60° to the right.

Phase shift 60° right

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EXAMPLE 2

Q Describe the shift in $y = \cos x + 3$, and give the midline and the range.

SOLUTION

What is outside moves the graph up or down, in the same direction as the sign. + 3 moves the whole graph 3 units up, so the midline is y = 3.

$3 - 1 = 2,\ 3 + 1 = 4$

The range is from 2 to 4.

Practice questions

1 Describe the shift in $y = \sin(x + 90°)$.

- **A** 90 degrees left
- **B** 90 degrees right
- **C** 45 degrees left
- **D** 45 degrees right

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2 Describe the shifts in $y = \sin(x - 30°) + 2$.

- **A** 30 degrees right and 2 units up
- **B** 30 degrees left and 2 units up
- **C** 30 degrees right and 2 units down
- **D** 30 degrees left and 2 units down

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3 Describe the shift in $y = \sin x - 4$.

- **A** 4 units down
- **B** 4 units up
- **C** 2 units down
- **D** 2 units up

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4 For $y = \cos x + 2$, what is the vertical shift and what is the midline?

- **A** 2 units down, midline y = -2
- **B** 2 units up, midline y = 2
- **C** 2 units up, midline y = -2
- **D** 2 units down, midline y = 2

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5 For $y = 3 \sin x + 1$, what is the amplitude and what is the midline?

- **A** Amplitude 1, midline y = 3
- **B** Amplitude 3, midline y = -1
- **C** Amplitude 3, midline y = 1
- **D** Amplitude 1, midline y = -3

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6

What is the range of $y = \sin x + 2$?

A From 1 to 3

B From -1 to 1

C From 0 to 2

D From 2 to 4

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7

Does a phase shift change the amplitude of a sine graph?

A Yes

B No

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8 Does a vertical shift change the maximum value of a cosine graph?

- **A** Yes
- **B** No

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9 A sine graph has its middle line at $y = -2$ and peaks at $y = 1$. What is the vertical shift and the amplitude?

- **A** 2 units down, amplitude 3
- **B** 2 units up, amplitude 3
- **C** 2 units down, amplitude 1
- **D** 1 unit down, amplitude 3

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10 **A cosine graph has amplitude 2, no phase shift, and midline $y = -1$. Write its equation.**

A $y = 2 \cos x - 1$

B $y = 2 \cos x + 1$

C $y = \cos x - 2$

D $y = 2 \sin x - 1$

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ANSWER KEY

1. 90 degrees left

2. 30 degrees right and 2 units up

3. 4 units down

4. 2 units up, midline $y = 2$

5. Amplitude 3, midline $y = 1$

6. From 1 to 3

7. No

8. Yes

9. 2 units down, amplitude 3

10. $y = 2 \cos x - 1$

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