MATHEMATICS · GRADES 10-12

Product rule

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you work out how fast a changing quantity grows when it is made of two changing parts, like the area of a garden that is getting longer and wider at the same time. A garden growing on both sides gains area faster than one growing on one side only, since both changes add to the growth. A shop whose price and number of customers are both changing faces the same question, since its revenue depends on both at once. Whenever two changing things multiply together, this is the rule that finds how fast their product is changing.



Rule

The derivative of $u \times v$ is $u' \times v + u \times v'$.

Differentiate each factor in turn, keep the other factor as it is, and add the two parts.

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Worked examples

EXAMPLE 1

Q Differentiate $x^2 \times x^3$, using the product rule.

SOLUTION

Let $u = x^2$ and $v = x^3$, so $u' = 2x$ and $v' = 3x^2$.

Product rule: $u' v + u v' = 2x \times x^3 + x^2 \times 3x^2$

That is $2x^4 + 3x^4 = 5x^4$.

EXAMPLE 2

Q Differentiate $x(x + 1)$.

SOLUTION

Let $u = x$ and $v = x + 1$, so $u' = 1$ and $v' = 1$.

Product rule: $1 \times (x + 1) + x \times 1 = x + 1 + x$

So the answer is $2x + 1$.

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Practice questions

1 State the product rule.

- **A** $(uv)' = u' v + u v'$
- **B** $(uv)' = u' v - u v'$
- **C** $(uv)' = u' v$
- **D** $(uv)' = u' + v'$

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2 Differentiate $x \times x^2$, using the product rule.

- **A** $2x$
- **B** $2x^2$
- **C** $3x^2$
- **D** $3x$

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3 Use the product rule on $x^2(x + 2)$.

- **A** $2x$
- **B** $2x^2 + 4x$
- **C** $3x^2 + 4x$
- **D** $3x^2 + 2x$

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4 Differentiate $4 \times x^3$ using the product rule.

- **A** $12x^2$
- **B** $4x^2$
- **C** $12x^3$
- **D** $3x^2$

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5 Differentiate $(x + 1)(x + 2)$.

- **A** $2x + 2$
- **B** $2x + 3$
- **C** $x + 3$
- **D** $2x + 1$

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6 Which of these is the correct first step of the product rule for $(x + 2)(x + 3)$?

- **A** $1 \times (x + 3) + (x + 2) \times 1$
- **B** 1×1
- **C** $(x + 3) + (x + 2)$
- **D** $2x + 5$

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7 Why can you not just multiply the derivatives?

- **A** Because the product changes through both factors at once
- **B** Because you multiply the two derivatives together
- **C** Because the product rule only works for sums
- **D** Because each factor changes on its own

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8 Differentiate $x \times x \times x$.

- **A** $3x^2$
- **B** x^2
- **C** $3x$
- **D** x^3

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9 Differentiate $x^2 \times x^3$, then find the value of the derivative at $x = 1$.

- **A** 4
- **B** 5
- **C** 6
- **D** 1

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10 Differentiate $(x^2 + 1)(x + 1)$.

- **A** $3x^2 + 2x + 1$
- **B** $2x^2 + 2x + 1$
- **C** $3x^2 + 2x$
- **D** $2x^2 + x + 1$

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ANSWER KEY

1. $(uv)' = u' v + u v'$

2. $3x^2$

3. $3x^2 + 4x$

4. $12x^2$

5. $2x + 3$

6. $1 \times (x + 3) + (x + 2) \times 1$

7. Because the product changes through both factors at once

8. $3x^2$

9. 5

10. $3x^2 + 2x + 1$

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