MATHEMATICS · GRADES 10-12

Pythagorean identities

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you find a missing trig value from one you already know, like a surveyor working out a slope angle from a single measurement on site. A surveyor who measures one side of a triangle can work out the rest without measuring the whole site, and a roofer can work out a roof pitch from a single rise measurement. A ramp builder checks the angle the same way, from one measurement instead of climbing up with a protractor. One known value is enough when the values are tied together this way.



Rule

$\sin^2 x + \cos^2 x = 1$.

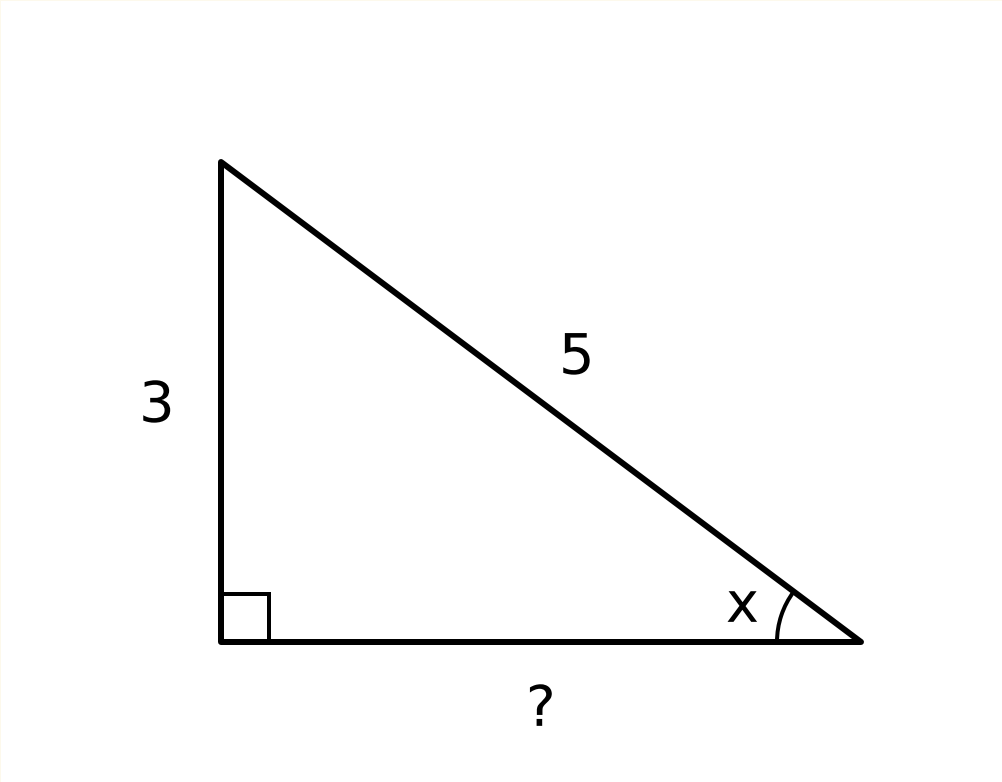
$1 + \tan^2 x = \sec^2 x$. $1 + \cot^2 x = \csc^2 x$. These hold for every angle x.

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Worked examples

EXAMPLE 1

Q If $\sin x = \frac{3}{5}$, find $\cos x$ for acute x.



SOLUTION

Use $\sin^2 x + \cos^2 x = 1$.

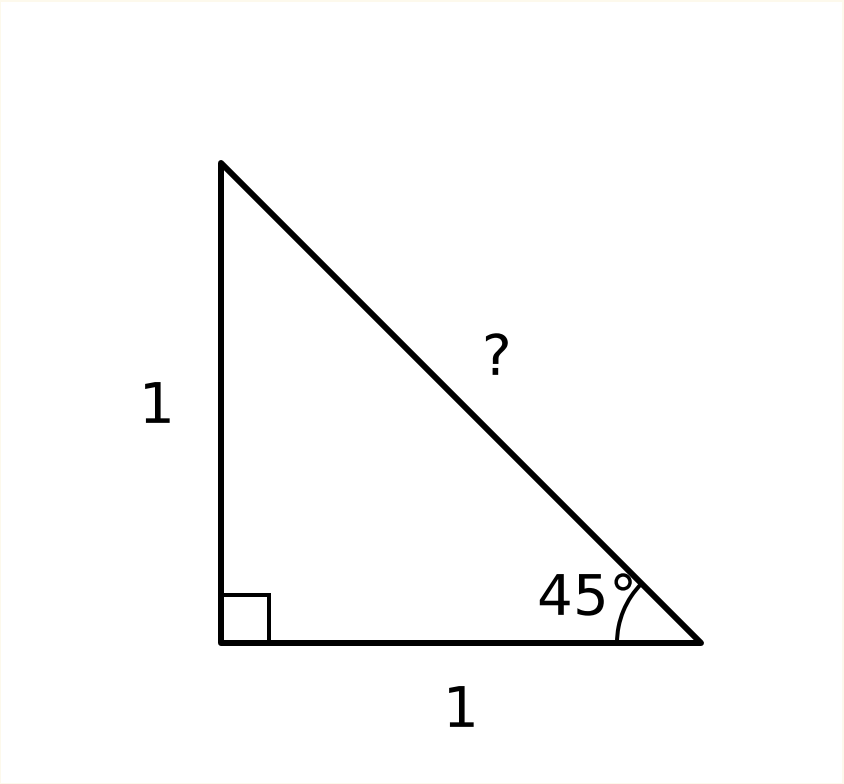
So $\cos^2 x = 1 - \frac{9}{25} = \frac{16}{25}$

For an acute angle, $\cos x$ is positive: $\cos x = \frac{4}{5}$

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EXAMPLE 2

Q Verify $1 + \tan^2 x = \sec^2 x$ for $x = 45$ degrees.



SOLUTION

Tan 45 degrees = 1, so $1 + 1^2 = 2$

Sec 45 degrees = $\sqrt{2}$, so $\sec^2 = 2$

Both sides are 2, so the identity holds here.

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Practice questions

1 If $\sin x = \frac{20}{29}$, find $\cos x$ for acute x.

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2 If $\cos x = \frac{5}{13}$, find $\sin x$ for acute x.

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3 If $\cos x = \frac{12}{13}$, find $\tan x$ for acute x.

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4 Verify $1 + \tan^2 x = \sec^2 x$ for $x = 60$ degrees.

- **A** Both sides are 2
- **B** Both sides are 3
- **C** Both sides are 4
- **D** Both sides are 5

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5 If $\tan x = \frac{3}{4}$, find $\sec x$ for acute x.

- **A** $\frac{5}{4}$
- **B** $\frac{4}{5}$
- **C** $\frac{3}{5}$
- **D** $\frac{4}{3}$

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6 Verify $\sin^2 x + \cos^2 x = 1$ for $x = 30$ degrees.

A $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

B $\frac{1}{4} + \frac{3}{4} = 1$

C $\frac{3}{4} + \frac{3}{4} = \frac{3}{2}$

D $\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$

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7 State the main Pythagorean identity.

A $\sin^2 x + \cos^2 x = 1$

B $\sin^2 x - \cos^2 x = 1$

C $\sin x + \cos x = 1$

D $\sin^2 x + \cos^2 x = 0$

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8 State the identity with tan and sec.

A $1 + \tan^2 x = \sec^2 x$

B $1 + \tan^2 x = \csc^2 x$

C $1 - \tan^2 x = \sec^2 x$

D $\tan^2 x + \sec^2 x = 1$

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9 If $\tan x = \frac{5}{12}$, find $\cos x$ for acute x.

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10 **Verify $1 + \cot^2 x = \csc^2 x$ for $x = 45$ degrees.**

A Both sides are 1

B Both sides are 2

C Both sides are 4

D Both sides are 3

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ANSWER KEY

1. $\frac{21}{29}$

2. $\frac{12}{13}$

3. $\frac{5}{12}$

4. Both sides are 4

5. $\frac{5}{4}$

6. $\frac{1}{4} + \frac{3}{4} = 1$

7. $\sin^2 x + \cos^2 x = 1$

8. $1 + \tan^2 x = \sec^2 x$

9. $\frac{12}{13}$

10. Both sides are 2

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