MATHEMATICS · GRADES 10-12

Quotient rule

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you work out how fast a ratio is changing, like how quickly your average speed changes when both distance and time are changing on a road trip. A driver who covers more distance but also takes more time may see the average speed rise or fall, depending on which changes faster. A shop watching its cost per customer faces the same question, since both the total cost and the number of customers are moving. Whenever a rate is made from two changing numbers, this is the rule that finds how fast the rate itself is changing.



Rule

The derivative of $\frac{u}{v}$ is $\frac{u' \times v - u \times v'}{v^2}$.

Differentiate the top, keep the bottom, then subtract the top times the derivative of the bottom, all over the bottom^2.

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Worked examples

EXAMPLE 1

Q Differentiate $\frac{x^2}{x}$, using the quotient rule.

SOLUTION

Let $u = x^2$ and $v = x$, so $u' = 2x$ and $v' = 1$.

Quotient rule: $\frac{(u'v - uv')}{v^2} = \frac{(2x \times x - x^2 \times 1)}{x^2}$

That is $\frac{(2x^2 - x^2)}{x^2} = 1$

EXAMPLE 2

Q Differentiate $\frac{1}{x}$.

SOLUTION

Let $u = 1$ and $v = x$, so $u' = 0$ and $v' = 1$.

Quotient rule: $\frac{(0 \times x - 1 \times 1)}{x^2} = -\frac{1}{x^2}$

So the answer is $-\frac{1}{x^2}$

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Practice questions

1 State the quotient rule.

A $\left(\frac{u}{v}\right)' = \frac{(u'\,v + u\,v')}{v^2}$

B $\left(\frac{u}{v}\right)' = \frac{(u'\,v - u\,v')}{v^2}$

C $\left(\frac{u}{v}\right)' = \frac{(u'\,v - u\,v')}{v}$

D $\left(\frac{u}{v}\right)' = \frac{(u\,v' - u'\,v)}{v^2}$

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2 Differentiate $\frac{x}{2}$.

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3 Differentiate $\frac{x+1}{x}$.

A $-\frac{1}{x^2}$

B $\frac{1}{x^2}$

C $-\frac{1}{x}$

D $-\frac{1}{x^3}$

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4 Differentiate $\frac{3}{x}$.

A $\frac{3}{x^2}$

B $-\frac{3}{x^2}$

C $-\frac{3}{x}$

D $-\frac{6}{x^2}$

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5 Differentiate $\frac{x^2}{2}$.

A x^2

B x

C $2x$

D 2

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6 Differentiate $\frac{2x}{x+1}$.

A $\frac{2}{(x+1)^2}$

B $\frac{2}{(x+1)}$

C $-\frac{2}{(x+1)^2}$

D $\frac{1}{(x+1)^2}$

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7 A student writes out the quotient rule but forgets the bottom of the answer. What goes on the bottom?

- **A** v
- **B** v^2
- **C** u^2
- **D** $2v$

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8 Differentiate $\frac{1}{x^2}$.

- **A** $-\frac{2}{x^3}$
- **B** $\frac{2}{x^3}$
- **C** $-\frac{2}{x^2}$
- **D** $-\frac{1}{x^3}$

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9 A student differentiates $\frac{x+1}{x}$ and gets $\frac{1}{x^2}$. What did they do wrong?

- **A** They forgot the minus sign
- **B** They forgot to square the bottom
- **C** They differentiated the top wrong
- **D** They used the product rule

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10 Differentiate $\frac{x^2+1}{x}$.

- **A** $1+\frac{1}{x^2}$
- **B** $1-\frac{1}{x^2}$
- **C** $1-\frac{2}{x^2}$
- **D** $1-\frac{1}{x}$

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ANSWER KEY

1. $\left(\frac{u}{v}\right)' = \frac{(u' v - u v')}{v^2}$

2. $\frac{1}{2}$

3. $-\frac{1}{x^2}$

4. $-\frac{3}{x^2}$

5. x

6. $\frac{2}{(x+1)^2}$

7. v^2

8. $-\frac{2}{x^3}$

9. They forgot the minus sign

10. $1 - \frac{1}{x^2}$

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