MATHEMATICS · GRADES 10-12

Related rates

Difficulty level: Advanced

Mathematics: Grades 10-12

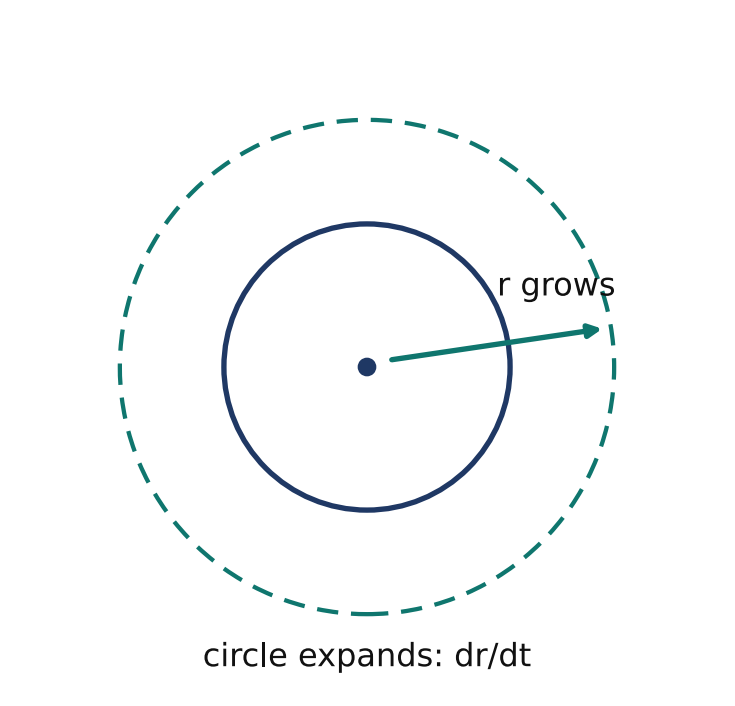
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Overview of the concept

This is how you work out how fast one thing is changing from how fast another is changing, like how fast a water level rises as a tank fills or how fast a shadow grows as you walk away from a lamp. A tank filling at a steady rate has a water level that rises faster when the tank is narrow, and a shadow grows faster the further you walk from the light. A ladder sliding down a wall and a balloon inflating are the same kind of problem, with two changing measurements tied together. When one rate is known, the other can be worked out from how they are linked.





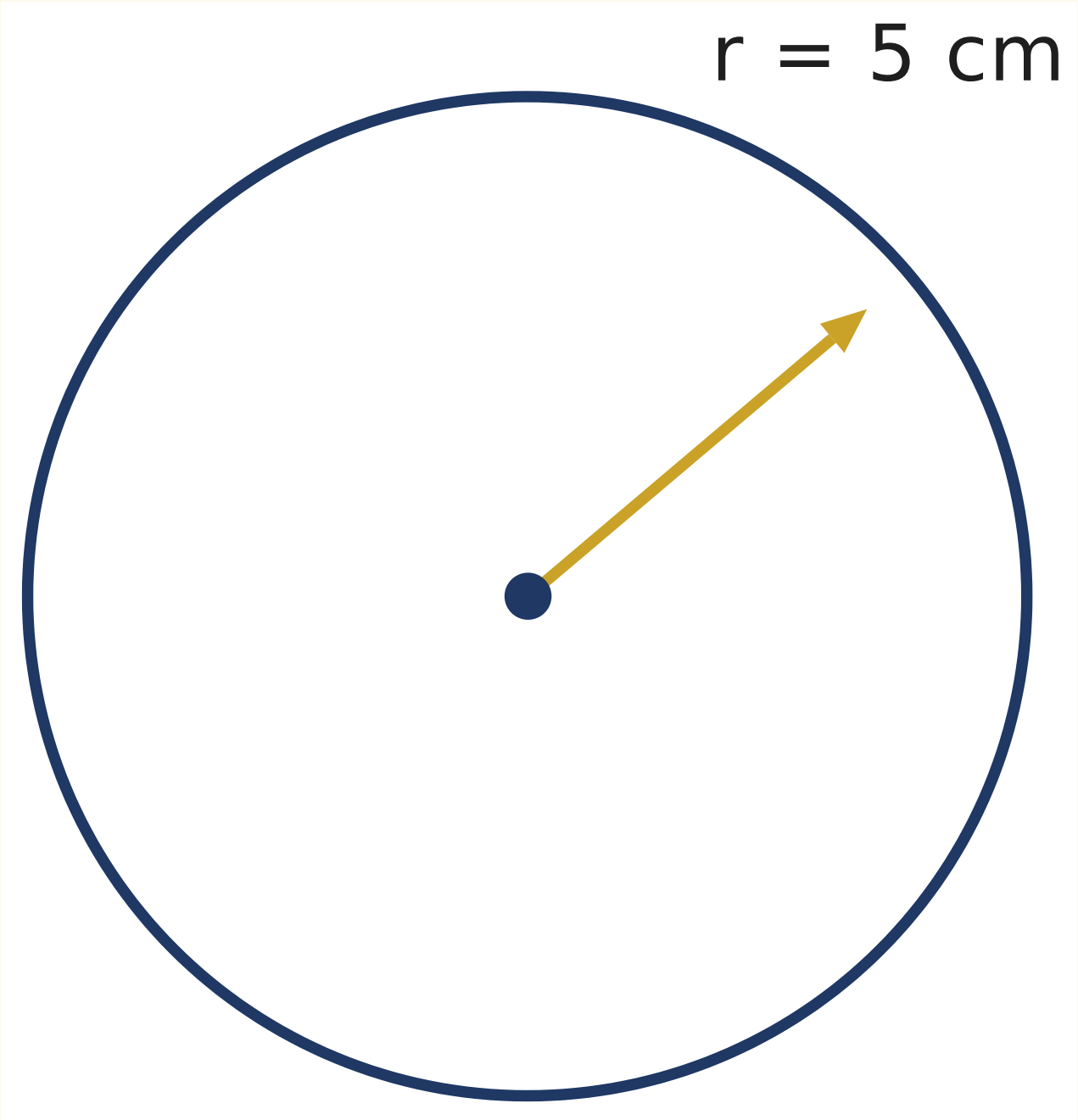
circle expands: dr/dt

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Worked examples

EXAMPLE 1

Q A balloon's radius grows at 2 cm per second. How fast does its area grow when r = 5 cm?



SOLUTION

Area = pi r^2. Differentiate with respect to time: dA/dt = 2 pi r × dr/dt

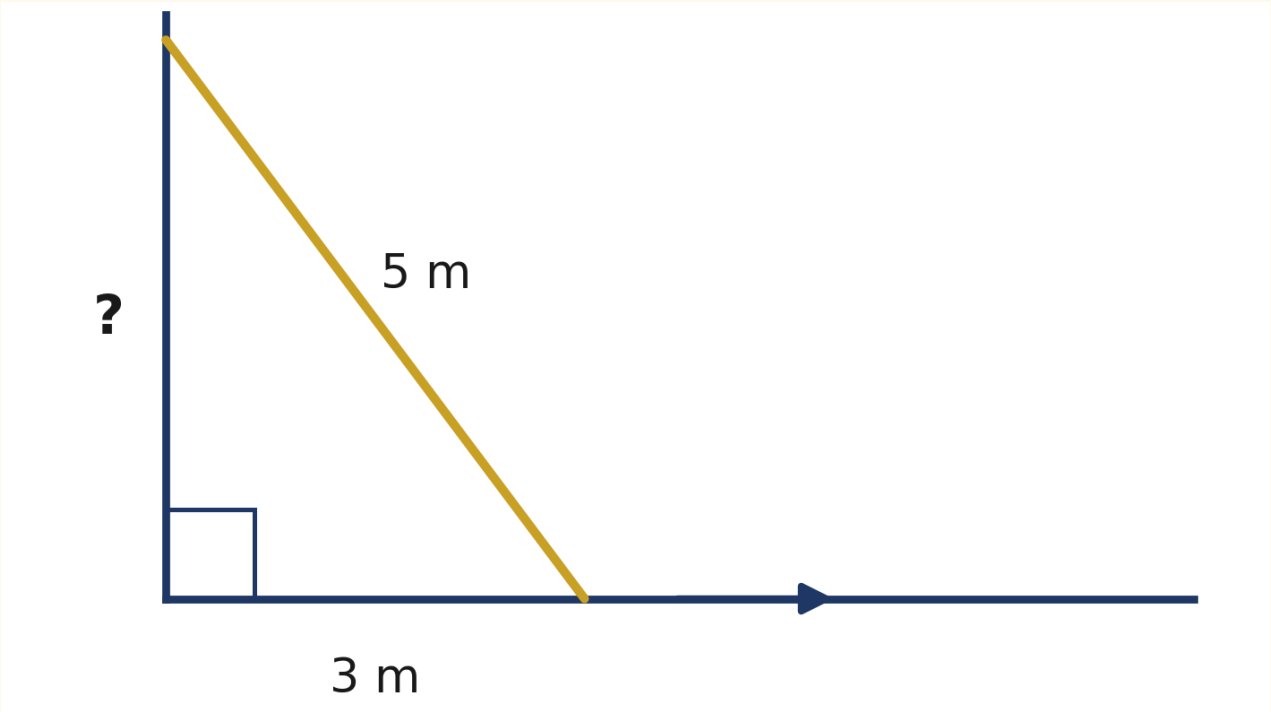
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Put r = 5 and dr/dt = 2: dA/dt = 2 pi × 5 × 2

Rate: 20 pi cm^2 per second

EXAMPLE 2

Q A ladder 5 m long slides: the bottom moves away at 1 m/s when the bottom is 3 m from the wall. How fast does the top fall?



SOLUTION

Let x be the bottom distance and y the top height: $x^2 + y^2 = 25$.

When x = 3, y = 4. Differentiate: 2x dx/dt + 2y dy/dt = 0

So $dy/dt = -\left(\frac{x}{y}\right) \times dx/dt = -\frac{3}{4}$ m/s: the top falls at 0.75 m/s.

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Practice questions

1 A circle's radius grows at 1 cm/s. How fast does its circumference grow?

- **A** 2 pi cm per second
- **B** pi cm per second
- **C** 4 pi cm per second
- **D** 2 cm per second

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2 A square's side grows at 2 cm/s. How fast does its area grow when the side is 3 cm?

- **A** 12 cm^2 per second
- **B** 6 cm^2 per second
- **C** 18 cm^2 per second
- **D** 4 cm^2 per second

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3 **A balloon's radius grows at 3 cm/s. How fast does its volume grow when r = 2 cm, using $V = (\frac{4}{3})$ pi r^3?**

A 48 pi cm^3 per second

B 16 pi cm^3 per second

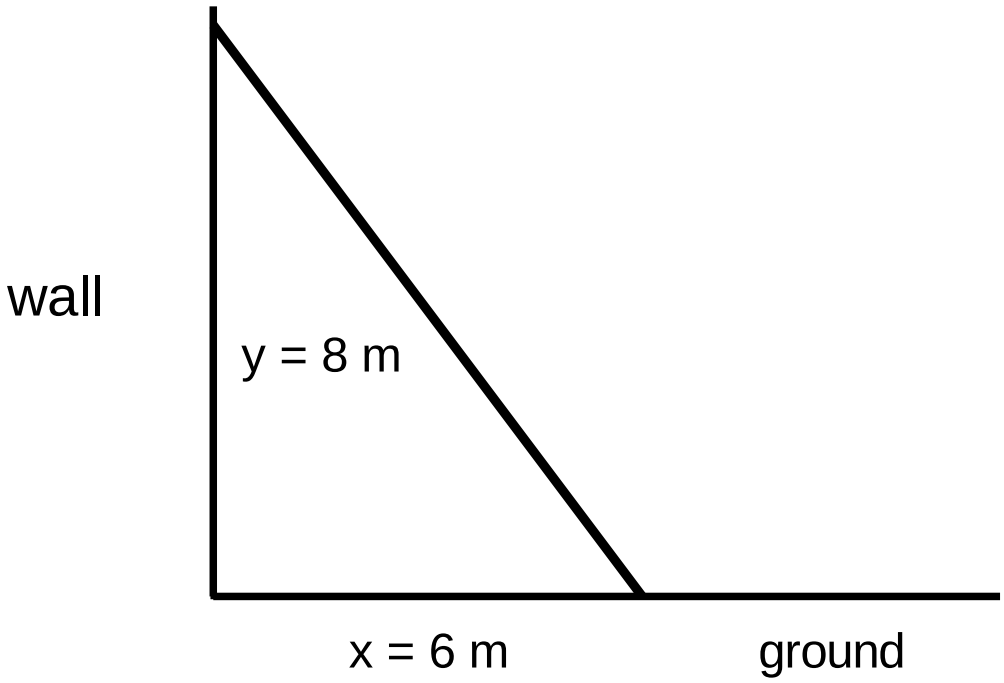
C 32 pi cm^3 per second

D 12 pi cm^3 per second

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4 **A 10 m ladder leans against a wall. The bottom of the ladder is 6 m from the wall, and the top is 8 m up the wall. The bottom slides away from the wall at 2 m/s. How fast does the top move?**



A -1.5 m/s

B -2 m/s

C 1.5 m/s

D -0.75 m/s

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5 A 5 m tall person walks away from a 10 m tall lamp post at 3 m/s. How fast does the tip of the person's shadow move?

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6 Water pours into a conical tank at 2 pi m^3 per minute. The tank is 6 m tall and the radius at the top is 3 m. How fast does the water level rise when the water is 2 m deep?

- **A** 2 m/min
- **B** 1 m/min
- **C** 4 m/min
- **D** 0.5 m/min

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7 A circle's area grows at 10 pi cm² per second when r = 5. How fast does the radius grow?

- **A** 1 cm per second
- **B** 2 cm per second
- **C** 5 cm per second
- **D** 10 cm per second

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8 A cube's side grows at 1 cm/s. How fast does its volume grow when the side is 2 cm?

- **A** 12 cm^3 per second
- **B** 6 cm^3 per second
- **C** 8 cm^3 per second
- **D** 24 cm^3 per second

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9 **A balloon's radius grows at 3 cm per second. How fast does its area grow when r = 4 cm?**

A 24 pi cm² per second

B 16 pi cm² per second

C 12 pi cm² per second

D 48 pi cm² per second

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10 **Two cars leave the same intersection at the same time. One drives north at 30 km/h and the other drives east at 40 km/h. How fast is the distance between them growing after 1 hour?**

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ANSWER KEY

1. 2 pi cm per second

2. 12 cm^2 per second

3. 48 pi cm^3 per second

4. -1.5 m/s

5. 6

6. 2 m/min

7. 1 cm per second

8. 12 cm^3 per second

9. 24 pi cm^2 per second

10. 50

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