MATHEMATICS · GRADES 10-12

Systems with no solution

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you spot when two plans can never agree, like two phone plans where one is always more expensive, because their lines run parallel and never meet. The two rules climb at the same rate but start from different places, so one stays ahead of the other forever. No pair of values can satisfy both at once, because the lines never cross. Recognising that early saves you from searching for an answer that is not there.



Rule

Systems with no solution: when two equations describe parallel lines, they never meet, so no point satisfies both.

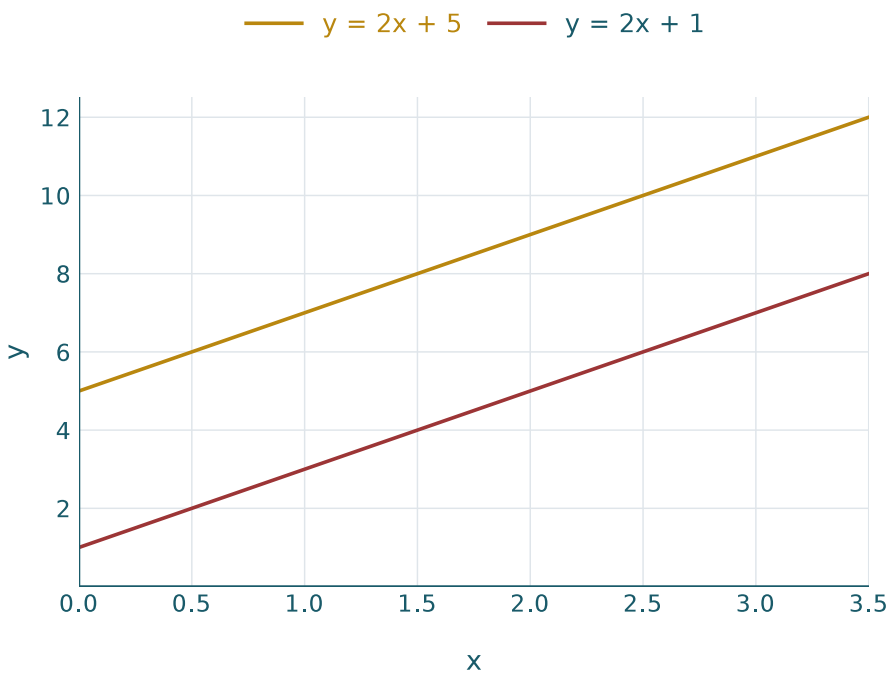
Solving gives a false statement, like 0 = 4.

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Worked examples

EXAMPLE 1

Q Show that $y = 2x + 1$ and $y = 2x + 5$ have no solution.



SOLUTION

Both lines have slope 2, but different intercepts, so they are parallel.

Setting them equal gives $2x + 1 = 2x + 5$, so $1 = 5$.

That is false, so there is no solution.

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EXAMPLE 2

Q Show that $x + y = 3$ and $2x + 2y = 10$ have no solution.



SOLUTION

Multiply the first equation by 2: $2x + 2y = 6$.

Compare with $2x + 2y = 10$: the left sides match but the right sides do not.

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Contradiction, so there is no solution.

Practice questions

1 The lines $y = 3x + 1$ and $y = 3x + 4$ never meet. Do they have a solution?

- **A** Yes
- **B** No

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2 Two phone plans cost $y = 30x + 20$ and $y = 30x + 45$, where x is the number of months. Can the two plans ever cost the same amount?

- **A** No, the lines are parallel and never meet
- **B** Yes, after 25 months
- **C** Yes, after 20 months
- **D** Yes, in the first month

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3 **Do $x + y = 5$ and $2x + 2y = 12$ have a solution?**

A Yes

B No

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4 **Do the lines $y = x + 2$ and $y = -x + 6$ have a solution?**

A Yes

B No

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Which pair of lines has no solution?

A $y = 2x + 1$ and $y = 2x + 5$

B $y = 2x + 1$ and $y = 3x + 1$

C $y = 2x + 1$ and $2y = 4x + 2$

D $y = x + 1$ and $y = -x + 1$

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Which statement shows that $x + y = 4$ and $2x + 2y = 12$ have no solution?

A. Multiplying the first equation by 2 gives $2x + 2y = 8$, which contradicts $2x + 2y = 12$

B. Multiplying the first equation by 2 gives $2x + 2y = 12$, which matches the second equation

C. Multiplying the first equation by 2 gives $2x + 2y = 6$, which contradicts $2x + 2y = 12$

D. Adding the equations gives $3x + 3y = 16$, so there is exactly one solution

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7 **Which statement shows that $y = 3x + 2$ and $y = 3x + 7$ have no solution?**

- **A** Both lines have slope 3 and the same intercept, so they are the same line
- **B** Both lines have slope 3 but different intercepts, so they are parallel; setting them equal gives $2 = 7$
- **C** Both lines have slope 2 but different intercepts, so they are parallel; setting them equal gives $2 = 7$
- **D** The lines meet where $3x + 2 = 3x + 7$, so there is one solution

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8 The lines $x + y = 5$ and $2x + 2y = 10$ lie exactly on top of each other. Do they have a solution?

- **A** No solution
- **B** Yes, one solution
- **C** Yes, infinitely many, the equations describe the same line
- **D** Yes, infinitely many, the lines are parallel

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9 What does $0 = 4$ at the end of solving a system mean?

- **A** No solution
- **B** One solution
- **C** Infinitely many solutions
- **D** The lines are the same line

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What do parallel lines have in common?

A The same intercept

B The same slope

C The same solution

D They meet at one point

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ANSWER KEY

1. No

2. No, the lines are parallel and never meet

3. No

4. Yes

5. $y = 2x + 1$ and $y = 2x + 5$

6. Multiplying the first equation by 2 gives $2x + 2y = 8$, which contradicts $2x + 2y = 12$

7. Both lines have slope 3 but different intercepts, so they are parallel; setting them equal gives $2 = 7$

8. Yes, infinitely many, the equations describe the same line

9. No solution

10. The same slope

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