MATHEMATICS · GRADES 10-12

Three-variable systems

Difficulty level: Advanced

Mathematics: Grades 10-12

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Overview of the concept

This is how you solve a problem with three unknowns, like the price of three different items from three different orders, by removing one unknown at a time. Three orders give you three facts, and each fact involves all three prices, so no price can be found on its own. Removing one unknown leaves a two unknown problem you already know how to solve, and then the third follows. That is how you untangle three linked prices, or any problem where three things have to be found together.



Rule

Three-variable systems: you need three equations for three unknowns, and you solve by removing one unknown at a time until you are left with a two-variable system you already know how to solve.

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Worked examples

EXAMPLE 1

Q Solve $x + y + z = 6$, $x + y = 4$, $z = 2$, by working backwards.

SOLUTION

The third equation already gives $z = 2$.

Put $z = 2$ into the first equation:

$x + y + 2 = 6$

$x + y = 4$

One solution family: $z = 2$ with any x, y adding to 4, for example (1, 3, 2).

EXAMPLE 2

Q Solve $x + y + z = 9$, $x + y = 5$, $y + z = 7$.

SOLUTION

Take the first equation minus the second: $z = 9 - 5 = 4$

From the third: $y = 7 - 4 = 3$

From the second: $x = 5 - 3 = 2$. Answer: (2, 3, 4)

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Practice questions

1 $x + y + z = 12, x + y = 7, y + z = 8$. Solve the system.

- **A** (4, 3, 5)
- **B** (3, 4, 5)
- **C** (4, 5, 3)
- **D** (5, 3, 4)

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2 $x + y + z = 10, x = 2, y = 3$. Solve the system.

- **A** (2, 3, 5)
- **B** (2, 3, 4)
- **C** (3, 2, 5)
- **D** (2, 5, 3)

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3 A system has three unknowns. How many equations do you need?

- **A** One
- **B** Two
- **C** Three
- **D** Four

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4 $x + y + z = 6$, $x + y = 4$, $x + z = 5$. Solve the system.

- **A** (3, 1, 2)
- **B** (2, 1, 3)
- **C** (3, 2, 1)
- **D** (1, 3, 2)

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5 $x + y + z = 15, x + y = 9, y + z = 11$. Solve the system.

- **A** (4, 5, 6)
- **B** (5, 4, 6)
- **C** (4, 6, 5)
- **D** (6, 5, 4)

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6 A three-variable system is given. What is the usual first step?

- **A** Eliminate one variable to get a two-variable system
- **B** Solve for x, y, and z all at once
- **C** Add all three equations together
- **D** Guess a value for x, then check it

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7 $x + y + z = 8, z = 1, x + z = 4$. Solve the system.

A (3, 4, 1)

B (4, 3, 1)

C (3, 1, 4)

D (1, 4, 3)

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8 $x + y + z = 20, x = 5, z = 7$. Solve the system.

A (5, 8, 7)

B (5, 7, 8)

C (8, 5, 7)

D (5, 8, 6)

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9 A three-variable system has a solution. What is it called?

- **A** An ordered pair
- **B** An ordered triple
- **C** A coordinate plane
- **D** A single number

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10 $x + y + z = 3$, $x = 1$, $y = 1$. Solve the system.

- **A** (1, 1, 1)
- **B** (1, 1, 2)
- **C** (1, 2, 1)
- **D** (2, 1, 1)

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ANSWER KEY

1. (4, 3, 5)

2. (2, 3, 5)

3. Three

4. (3, 1, 2)

5. (4, 5, 6)

6. Eliminate one variable to get a two-variable system

7. (3, 4, 1)

8. (5, 8, 7)

9. An ordered triple

10. (1, 1, 1)

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